

Due: Friday, February 13 (start of class)

- For this homework set, you should work in a team of 2 or 3 and turn in a *single* paper for the entire team. See further details on the About→Homework Sets page of the course web site. For *Mathematica* work here, turn in printed pages—just what’s directly relevant. You may, and in fact are encouraged, to place associated written work directly onto such printed pages (provided it’s neat and easy to find).
 - See further instructions about homework **format** on the web site.
1. (a) Using a relevant formula, and *without* actually enumerating the subsets involved, calculate:
 - i. The number of 1-element subsets of a 10-element set.
 - ii. The number of 2-element subsets of a 10-element set.
 - iii. The number of 4-element subsets of a 10-element set.
 - iv. The number of 6-element subsets of a 10-element set.
 - v. The number of 8-element subsets of a 10-element set.
 - vi. The number of 10-element subsets of a 10-element set.
 - vii. The number of subsets of a 10-element set.
 (b) Now, for $k = 0, 1, 2, \dots, 10$:
 - exhibit all the k -element subsets of a 10-element set;
 - actually count the number of such k -element subsets (you may use `Card`); and
 - verify that this number is $\binom{10}{k}$.
 Use relevant functions described in `SetsAndFunctions.nb` or `Combinatorics.nb` to form the sets of these subsets.
 2. Do Review Exercise 1.8.17, page 22 of the textbook.
 3. Do the following exercise from notebook `SetsAndFunctions.nb`: In terms of functions already discussed above, define a function `SymmetricDifference` that finds the symmetric difference of two sets. Your definition should take the form `SymmetricDifference[x_, y_] := ...`, and the result of `SymmetricDifference[setA, setB]` for two sets `setA` and `setB` should be their symmetric difference.
 4. (a) For subsets A and B of a set X , express the characteristic function $c_{A\Delta B}$ of the symmetric difference of A and B in X in terms of the characteristic functions c_A and c_B of A and B , respectively, in X .
 (b) Prove that symmetric difference is associative, that is, for all subsets A, B, C of a set X , we have $A\Delta(B\Delta C) = (A\Delta B)\Delta C$. Use the result of (a) along with the fact that two subsets of X are equal when their characteristic functions are the same.
 5. Use relevant functions described in `SetsAndFunctions.nb` or `Combinatorics.nb` to form the sets of injections described. In (c), use `Card` to count the number of injections.
 - (a) Exhibit all injections from $\{1, 2\}$ to $\{1, 2, 3, 4, 5\}$.
 - (b) Exhibit all injections from $\{1, 2, 3, 4, 5, 6\}$ to $\{1, 2, 3, 4, 5\}$. Account for the result.
 - (c) For $k = 1, 2, 3, 4, 5, 6$, calculate the number of injections from a k -element set to a 5-element set and verify that this number is $P(5, k)$.