Math 236 work for May 7, 2001

Exercise 1. Let B be the ordered basis $(\overrightarrow{b_1}, \overrightarrow{b_2}, \overrightarrow{b_3})$ of $\mathbb{R}^2$ consisting of the vectors

$$\overrightarrow{b_1} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \qquad \overrightarrow{b_2} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \qquad \overrightarrow{b_3} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Suppose $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ is the linear transformation such that

$$T(\overrightarrow{b_1}) = \vec{0}, \qquad T(\overrightarrow{b_2}) = \overrightarrow{b_2}, \qquad T(\overrightarrow{b_3}) = 2\overrightarrow{b_3}.$$

(a) Find the matrix $[T]_B$ of T with respect to the ordered basis B.

(b) If $\vec{x} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$, then use $[T]_B$ to calculate $T(\vec{x})$.

Exercise 2. Let B and T be as in Exercise 1.

(a) Find the *standard* matrix $[T]$ of T.

(b) If again $\vec{x} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$, then use the standard matrix $[T]$ to calculate $T(\vec{x})$.

(c) If you have both $[T]_B$ and $[T]$, which way to calculate $T(\vec{x})$ in this situation is easier—using $[T]_B$ or $[T]$?