Math 236 work for May 4, 2001

Exercise 1. Let $\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

(a) For the ordered basis $B = (\overrightarrow{b_1}, \overrightarrow{b_2})$ of $\mathbb{R}^2$ given by $\overrightarrow{b_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\overrightarrow{b_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, find the coordinate vector $[\vec{x}]_B$ of $\vec{x}$ with respect to B.

(b) For the ordered basis $B' = (\overrightarrow{b_2}, \overrightarrow{b_1})$ of $\mathbb{R}^2$ given by the same vectors as in (a) but in the opposite order, find the coordinate vector $[\vec{x}]_{B'}$ of $\vec{x}$ with respect to B'.

(c) For the standard ordered basis $E = (\overrightarrow{e_1}, \overrightarrow{e_2})$ of $\mathbb{R}^2$, find the coordinate vector $[\vec{x}]_E$ of $\vec{x}$ with respect to E.

Exercise 2. What is the coordinate vector $[\vec{x}]_E$ of an arbitrary vector $\vec{x} \in \mathbb{R}^n$ with respect to the standard ordered basis $E = (\overrightarrow{e_1}, \overrightarrow{e_2}, \ldots, \overrightarrow{e_n})$ of $\mathbb{R}^n$?

Exercise 3. Let $B = (\overrightarrow{b_1}, \overrightarrow{b_2}, \ldots, \overrightarrow{b_n})$ be an arbitrary ordered basis of $\mathbb{R}^n$.

(a) What is the coordinate vector $[\overrightarrow{b_1}]_B$?

(b) For each $j = 1, 2, \ldots, n$, what is the coordinate vector $[\overrightarrow{b_j}]_B$?

Exercise 4. Again let $B = (\overrightarrow{b_1}, \overrightarrow{b_2})$ be the ordered basis of $\mathbb{R}^2$ given by $\overrightarrow{b_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\overrightarrow{b_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(a) If $\vec{x} \in \mathbb{R}^2$ has coordinate vector $[\vec{x}]_B = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$ with respect to B, then what is $\vec{x}$?

(b) What is the change-of-basis matrix S from B to the standard ordered basis $E = (\overrightarrow{e_1}, \overrightarrow{e_2})$?

(c) Use the change-of-basis matrix S somehow to find the coordinate vector $[\vec{x}]_B$ of $\vec{x}$ with respect to B for $\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.