Math 236 work for May 1, 2001

Exercise 1. Again let $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. You found that the eigenvalues of A are

$$\lambda_1 = 2, \qquad \lambda_2 = 3.$$

Also, for arbitrary λ, you calculated:

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 0 & -2 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

(a) Find bases of the eigenspaces E_{λ_1} and E_{λ_2} of A associated with these eigenvalues. Then tell the dimension of each eigenspace.

(b) Use your answer to (a) to describe all eigenvectors of A associated with each eigenvalue.

Exercise 2. Let A be the same matrix as in Exercise 1.

(a) Explain how you can tell now that A is diagonalizable.

(b) Find an invertible matrix S and a diagonal matrix S with $S^{-1}AS = D$.

(c) Use your answer to (b) to calculate A^3 (without actually multiplying A by itself three times).