Math 236 work for April 27, 2001

Exercise 1. Let $A = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$.

(a) Directly use the definitions involved to verify that $\lambda_1 = 2$ is an eigenvalue of A and that $\vec{v_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector associated with λ_1.

(b) Verify that $\vec{w_1} = \begin{bmatrix} -9 \\ 9 \end{bmatrix}$ is another eigenvector associated with that same eigenvalue λ_1.

(c) Verify that $\lambda_2 = 5$ is another eigenvalue of A and that $\vec{v_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is an eigenvector associated with A.

Exercise 2. If $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, verify that $\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector of A and find the associated eigenvalue λ.

Exercise 3. Why did we explicitly exclude $\vec{0}$ from being considered an eigenvector of a matrix A? (*Hint:* If it were considered an eigenvector, with what eigenvalue would it be associated?)

Exercise 4. If $A = I_n$, what are the eigenvalues of A and for each eigenvalue what are the associated eigenvectors?

Exercise 5. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be orthogonal projection onto a plane V that passes through the origin. Thinking *geometrically*:

(a) Show that each $\vec{v} \in V$ is an eigenvector of T and determine with what eigenvalue it is associated.

(b) Show that each $\vec{v} \in V^{\perp}$ is an eigenvector of T and determine with what eigenvalue it is associated.

Exercise 6. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be rotation of the plane around the origin counterclockwise through an angle α with $0 < \alpha < \pi$. Explain *geometrically* why T has no eigenvectors (and hence no eigenvalues).

Theorem 1. *Scalar λ is an eigenvalue of square matrix A if and only if $\det(\lambda I - A) = 0$.*

Exercise 7. Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Use the preceding theorem to find all eigenvalues of A.

Exercise 8. Let $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Use the preceding theorem to find all eigenvalues of A.

Exercise 9. Again let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. We found that the eigenvalues of A are

$$\lambda_1 = 2, \qquad \lambda_2 = 3.$$

Also, for arbitrary λ, we calculated:

$$\lambda I - A = \begin{bmatrix} \lambda - 2 & 0 & 2 \\ 0 & \lambda - 3 & 0 \\ 0 & 0 & \lambda - 3 \end{bmatrix}$$

(a) Find bases of the eigenspaces E_{λ_1} and E_{λ_2} of A associated with these eigenvalues. Then tell the dimension of each eigenspace.

(b) Use your answer to (a) to describe all eigenvectors of A associated with each eigenvalue.