Math 236                  Problem Set 10                  Spring, 2001

Due: Friday, May 4

Contrary to any “Do not use technology” instructions in the text, you may—and probably should—use Mathematica to do row reductions, solve linear systems, find bases of null spaces, etc.

As usual, include printouts of all Mathematica work whose results you used!

1. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the reflection of the $x_1x_2$-plane in the line $L$ having equation $x_2 = x_1$. Arguing geometrically—that is, do not form any matrices or calculate any determinants or solve any linear systems—find all eigenvectors and associated eigenvalues of $T$.

2. Suppose $\lambda$ is an eigenvalue of an $n \times n$ matrix $A$. Deduce that $\lambda^2$ is an eigenvalue of $A^2$. Then determine all the eigenvectors of $A^2$ with which $\lambda^2$ is associated. Use the definitions involved—but no determinants!

3. Do page 301, Exercise 36. In (b), use Mathematica to plot points on the trajectory.

4. Do page 311, Exercise 28 (a)–(b). Also, determine what happens in the long term—how many families shop each week at Wipf’s and how many at Migros (I suggest you use Mathematica to do the calculations for that). (For the definition of regular transition matrix, see exercise 25.)

5. In each part, also state what the algebraic and geometric multiplicities of each eigenvalue are.

   (a) Do page 324, Exercise 14.

   (b) Do page 324, Exercise 12.