This is an extract from the 3rd Spring 2006 mid-semester exam. It consists only of those questions relevant to the Spring 2007 Exam 3.

1. (2 × 6% = 12%) The parts of this question are not related.
   (a) Find the exact value of the repeating decimal $0.\overline{15} = 0.151515\ldots$ as a rational number.

2. (4 × 5% = 20%) Does the series converge? Why or why not? (Name any test you use and show that the conditions needed for that test actually hold.)
   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$
   (b) $\sum_{n=1}^{\infty} \frac{3^n}{10^n}$
   (c) $\sum_{n=1}^{\infty} \frac{n^2 + 1}{7n^2 + 3n - 5}$
   (d) $\sum_{n=1}^{\infty} \frac{\sqrt{n^4 + 1}}{n^4 + 3n + 5}$

3. (a) (4%) Approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$ by the sum of its first three terms. Round your answer to six decimal places.
   (b) (8%) Without using any value for the actual sum of the series, find an upper bound for the error involved in that approximation. Round your answer to six decimal places.

4. (2 × 8% = 16%) The parts of this question are not related.
   (a) Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x - 3)^n}{n \cdot 2^n}$.
   (b) Find the first 4 terms in the Taylor series for $f(x) = \sin x$ about $\frac{\pi}{4}$.

5. (2 × 5% = 10%)
   (a) Find a power series representation for the function $\frac{1}{1 + x^4}$.
       Use summation (\(\sum\)) notation to express your answer.
   (b) Use (a) to express $\int_0^{1/2} \frac{1}{1 + x^4} \, dx$ as the sum of an infinite series.