Math 132  Exam 1  March 1, 2006
6:00-7:30 p.m.

Instructions

• Turn off all cell phones and watch alarms!
  Put away cell phones, iPods, etc.

• There are seven (7) questions.

• Do all work in this exam booklet. You may continue work to the backs of
  pages and the blank page at the end, but if you do so indicate where.

• Do not use any other paper except this exam booklet and the one-page “cheat
  sheet” that you prepared.

• Organize your work in an unambiguous order. Show all necessary steps.

• Answers given without supporting work may receive 0 credit!

• Do not write anything in the table below.

• Be prepared to show your UMass ID card when you hand in your exam booklet.

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<th>QUESTION</th>
<th>PER CENT</th>
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1. \((2 \times 8\% = 16\%)\) Let \(f(x) = x^2 + 10\).

(a) Approximate the definite integral \(\int_{2}^{10} f(x)\, dx\) by the Riemann sum that uses 4 subintervals and \textbf{right} endpoints as the sample points. Show the individual terms of the sum before you calculate the value of the sum.

(b) On the graph below, draw the rectangles that correspond to the Riemann sum in (a).
2. (15%) Water is being pumped into a storage tank at a rate of

\[ r(t) = 60 e^{-t/15} \text{ liters per second} \]

where time \( t \) is measured in seconds. How much water is pumped into the tank during the entire time interval \( 30 \leq t \leq 40 \)? Use a definite integral to arrive at your answer.
3. (15%) Sketch the region enclosed by the curves $y = x$ and $y^2 = x + 2$ and then determine its area.
4. (2 × 8% = 16%) Use appropriate substitutions to evaluate the following integrals. For each, clearly indicate the substitution you are using. Show all your work. (You may use your calculator to check your answer.)

(a) \[ \int \frac{t^2}{\sqrt{t} - 1} \, dt \]

(b) \[ \int_0^1 \frac{\arctan x}{x^2 + 1} \, dx \]  
(Note: \( \arctan \) is the inverse function of \( \tan x \).)
5. \( (2 \times 7\% = 14\%) \) A particle moves along the \( x \)-axis with velocity \( v(t) = t^3 - 3t \) for \( 0 \leq t \leq 5 \). The particle is at the origin at time 0.

(a) What is the \textbf{total distance} that the particle traverses—the “odometer distance”—between times \( t = 0 \) and \( t = 5 \)?

(b) What is the \( x \)-coordinate of the particle at time \( t = 5 \)?
6. (2 × 6% = 12%) The plane region in the first quadrant that is enclosed by the curves \( y = 0, \ y = \sin x, \) and \( x = \pi \) is rotated around the line \( y = 1 \) to form a solid.

(a) Find a formula for the area \( A(x) \) of cross-sections of the solid formed by planes perpendicular to the axis of rotation.

(b) Express the volume of the solid as a definite integral. Do not evaluate that integral!
7. (2 × 6% = 12%) Find the following and *justify your answers.*

(a) The value of

\[ \int_0^1 \frac{d}{dx} \left( \frac{e^{x^2+1}}{x^2 + 4} \right) \, dx. \]

(b) For \( x > 0, \) the *derivative* of the function

\[ F(x) = \int_e^{\sqrt{x}} \ln t \, dt. \]
This page left blank for additional work.