1. (a) [4%] First, find critical points of \( f \) [without expanding \( f'(x) \)]:

\[
\begin{align*}
    f'(x) &= 0 \iff x(x - 2)^3 = 0 \\
    &\iff x = 0 \text{ or } x = 2
\end{align*}
\]

Now proceed in either of two ways:

**Method 1: Sample values.** The derivative is continuous, so it suffices to test individual points elsewhere:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -1 )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) )</td>
<td>27</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>sign(( f'(x) ))</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

Hence:

- \( f \) is increasing for \( x < 0 \) and for \( x > 2 \), that is, on \((-\infty, 0)\) and \((2, \infty)\); and
- \( f \) is decreasing for \( 0 < x < 2 \), that is, on \((0, 2)\).

**Method 2: Use inequalities.** From \( f'(x) = x(x - 2)^3 \):

- \( x < 0 \implies x < 0 \) and \( x - 2 < 0 \implies x < 0 \) and \( (x - 2)^3 < 0 \implies f'(x) > 0 \implies f \) is increasing;
- \( 0 < x < 2 \implies x > 0 \) and \( x - 2 < 0 \implies x > 0 \) and \( (x - 2)^3 < 0 \implies f'(x) < 0 \implies f \) is decreasing; and
- \( x > 2 \implies x > 0 \) and \( x - 2 > 0 \implies f'(x) > 0 \implies f \) is increasing.

(b) [4%]

\[
\begin{align*}
f''(x) &= (f'(x))' = x(3(x - 2)^2) + (x - 2)^3(1) \\
&= (x - 2)^2 (3x + (x - 2)) = (x - 2)^2 (4x - 2) = 2(2x - 1)(x - 2)^2
\end{align*}
\]

Thus the critical numbers of \( f' \) are \( x = 1/2 \) and \( x = 2 \). Again, proceed in either of two ways:

**Method 1: Sample values.**

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f''(x) )</td>
<td>-8</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>sign(( f''(x) ))</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

Hence:

- \( f \) is concave down for \( x < 1/2 \); and
- \( f \) is concave up for \( x > 1/2 \) (or, you could say, for \( 1/2 < x < 2 \) and for \( x > 2 \)).

**Method 2: Use inequalities.** Since \( 2(x - 2)^2 \geq 0 \), the sign of \( f''(x) \) is the same as that of \( 2x - 1 \).

- \( x < 1/2 \implies 2x - 1 < 0 \implies f''(x) < 0 \implies f \) is concave down;
- \( 1/2 < x < 2 \implies 2x - 1 > 0 \implies f''(x) > 0 \implies f \) is concave up; and
- \( x > 2 \implies 2x - 1 > 0 \implies f''(x) > 0 \implies f \) is concave up.

(c) [4%] From (b), \( f \) has only one inflection point, namely, at \( x = 1/2 \).

(The concavity does not change at \( x = 2 \), so there is no inflection point there!)

(d) [4%] From (a) and the First Derivative Test, or from (a) and (b) and the Second Derivative Test:

- \( f \) has a local maximum at \( x = 0 \); and
- \( f \) has a local minimum at \( x = 2 \).
2. [16%] Variables: Let

\[ t = \text{time (years) after start}, \]
\[ Q(t) = \text{mass of UMa-2008 at time } t. \]

Model: For some constant \( k \), \( Q'(t) = k \) \( Q(t) \), that is,

\[ Q(t) = Q(0) e^{kt}. \]

Given: \( Q(2) = 0.85 \) \( Q(0) \).

To solve for \( t \): \( Q(t) = 0.10 \) \( Q(0) \).

Find \( k \) first: Use the given relation \( Q(2) = 0.85 \) \( Q(0) \):

\[
Q(0) e^{k(2)} = 0.85 Q(0) \\
e^{2k} = 0.85 \\
2k = \ln 0.85 \\
k = \frac{\ln 0.85}{2} \quad \text{(Note: } k < 0) 
\]

Then

\[ Q(t) = Q(0) e^{\left(\frac{\ln 0.85}{2}\right)t}. \]

Finally, solve \( Q(t) = 0.10 \) \( Q(0) \) for \( t \):

\[
Q(0) e^{\left(\frac{\ln 0.85}{2}\right)t} = 0.10 Q(0) \\
e^{\left(\frac{\ln 0.85}{2}\right)t} = 0.10 \\
\left(\frac{\ln 0.85}{2}\right)t = \ln 0.10 \\
t = \frac{2 \ln 0.10}{\ln 0.85} \approx 28.336.
\]

Answer: About \( \underline{28.336 \text{ years}} \).

(If you interpret the question as asking how much longer it takes after the initial 2-years' decay, then the answer would be, instead, about 26.336 years.)
3. (a) [6%] The quotient is “0/0-form” because
\[
\lim_{x \to 0} (2e^x - 2) = 2e^0 - 2 = 0, \quad \lim_{x \to 0} x = 0.
\]
Then:
\[
\lim_{x \to 0} \frac{2e^x - 2}{x} = \lim_{x \to 0} \frac{d(2e^x - 2)/dx}{d(x)/dx} \quad \text{(L'Hospital's Rule)}
\]
\[
= \lim_{x \to 0} \frac{2e^x}{1} = 2e^0 = \boxed{2}.
\]

(b) [5%] L'Hospital's Rule does \textit{not} apply here, because \(\lim_{x \to 0} x + \cos x = 0 + \cos 0 = 1 \neq 0\). However, by Direct Substitution (or the ordinary Quotient Rule for limits):
\[
\lim_{x \to 0} \frac{x + \sin x}{x + \cos x} = \frac{0 + \sin 0}{0 + \cos 0} = \frac{0}{1} = \boxed{0}.
\]

(c) [5%] The quotient is “\(\infty/\infty\)-form” because
\[
\lim_{x \to \infty} (1 + x) = \infty, \quad \lim_{x \to \infty} \frac{1}{x} = 0.
\]
Let
\[
y = (1 + x)^{1/x}
\]
so that
\[
\ln y = \frac{\ln(1 + x)}{x}
\]
Now
\[
\lim_{x \to \infty} \ln(1 + x) = \infty, \quad \text{and} \quad \lim_{x \to \infty} x = \infty,
\]
so that the quotient \(\ln y = \frac{\ln(1 + x)}{x}\) is “\(\infty/\infty\)-form”. Then
\[
\lim_{x \to \infty} (\ln y) = \lim_{x \to \infty} \frac{\ln(1 + x)}{x} = \lim_{x \to \infty} \frac{d(\ln(1 + x))/dx}{d(x)/dx} \quad \text{(by L'Hospital's Rule)}
\]
\[
= \lim_{x \to \infty} \frac{1/(1 + x)}{1} = \lim_{x \to \infty} \frac{1}{1 + x} = 0.
\]
Finally,
\[
\lim_{x \to \infty} (1 + x)^{1/x} = \lim_{x \to \infty} y = \lim_{x \to \infty} e^{\ln y} = \lim_{x \to \infty} e^{\ln y} = e^{\lim_{x \to \infty} \ln y} = e^0 = \boxed{1}.
\]
4. (a) [10%] Let $a = 0$.

\[ f(x) = e^x \implies f(a) = e^0 = 1, \]

\[ f'(x) = e^x \implies f'(a) = e^0 = 1 \]

Then:

\[ L(x) = f(a) + f'(a)(x - a) \]

\[ = 1 + 1(x - 0) = 1 + x \]

(b) [6%]

\[ e^{-0.2} = f(-0.2) \approx L(-0.2) \]

\[ = 1 + (-0.2) = 0.8 \]

This is an exact decimal, so no further rounding is needed. Thus the approximation is:

\[ e^{-0.2} \approx 0.8 \]

[Note: It would be misleading to write that $L(-0.2) \approx 0.8$, since the value of $L(-0.2)$ is exactly 0.8. And it would be wrong to write that $e^{-0.2} = 0.8$, since, as the TI-89 shows, $e^{-0.2} \approx 0.818731$, that is, $e^{-0.2} \approx 0.819$ when rounded to 3 decimal places.]

5. (a) [6%] If $f$ is continuous on the closed interval $[a, b]$ and differentiable on the open interval $(a, b)$, then...

\[ \text{there is some } c \text{ in the open interval } (a, b) \text{ at which } \frac{f(b) - f(a)}{b - a} = f'(c). \]

(b) [10%] The function \( f(x) = \ln x \) is differentiable and continuous for \( x > 0 \) and hence on \( [a, b] = [1, 3] \). Moreover,

\[ f'(x) = \frac{1}{x} \]

for all \( x > 0 \). By the Mean Value Theorem, there is some \( c \) in \((1, 3)\) at which

\[ \frac{f(3) - f(1)}{3 - 1} = f'(c), \]

that is,

\[ \frac{\ln 3 - \ln 1}{2} = \frac{1}{c}, \]

and since \( \ln 1 = 0 \),

\[ \ln 3 = 2 \cdot \frac{1}{c}. \]

Now \( c < 3 \), so that \( \frac{1}{c} > \frac{1}{3} \). Hence \( \ln 3 > \frac{2}{3} \).
6. [16%] Let

\[ t = \text{time (sec)}, \]
\[ y = \text{height (ft) of rocket at time } t \]
\[ \theta = \text{angle of elevation at time } t \]
\[ \text{of rocket from camera} \]
\[ z = \text{distance (ft) at time } t \]
\[ \text{between camera and rocket}. \]

Given:

\[ \frac{dy}{dt} = 500. \]

To find: \[ \frac{d\theta}{dt} \bigg|_{y=3000}. \]

**Method 1:** Use the relation: \[ \tan \theta = \frac{y}{5000} \]

Take \( \frac{d}{dt} \) of the preceding relation:

\[
\begin{align*}
(\sec^2 \theta) \frac{d\theta}{dt} &= \frac{dy/\ dt}{5000}, \\
(\sec^2 \theta) \frac{d\theta}{dt} &= \frac{500}{5000} = \frac{1}{10}, \\
\frac{d\theta}{dt} &= \frac{1}{10} \cos^2 \theta.
\end{align*}
\]

In general,

\[ \cos \theta = \frac{5000}{z}. \]

Now when \( y = 3000: \)

\[ z = \sqrt{(5000)^2 + (3000)^2} = \sqrt{34,000,000} = 1000\sqrt{34} \]

so that

\[ \cos \theta = \frac{5000}{1000\sqrt{34}} = \frac{5}{\sqrt{34}} \]

Hence

\[
\frac{d\theta}{dt} \bigg|_{y=3000} = \frac{1}{10} \left( \frac{5}{\sqrt{34}} \right)^2 = \frac{1}{10} \frac{25}{34} = \frac{5}{68} \\
\approx 0.0735294 \approx 0.074
\]

**Answer:** The camera rotates at a rate of approximately \( 0.074/\text{sec} \), i.e., 0.074 radians/sec.

(This is approximately 4.23°/sec.)
Method 2: Use the relation $\theta = \arctan \frac{y}{5000}$.

Take $\frac{d}{dt}$:

\[
\frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{y}{5000}\right)^2} \frac{1}{5000} \frac{dy}{dt} \quad \text{(Chain Rule)}
\]

\[
= \frac{(5000)^2}{(5000)^2 + y^2} \cdot \frac{1}{5000} \cdot 500 \quad \text{(from given value of } \frac{dy}{dt})
\]

\[
= \frac{1}{10} \frac{(5000)^2}{(5000)^2 + y^2}
\]

so that

\[
\left. \frac{d\theta}{dt} \right|_{y=3000} = \frac{1}{10} \frac{(5000)^2}{(5000)^2 + (3000)^2} = \frac{5}{68} \approx 0.074
\]