**Math 235 Practice Final Exam** **Spring 2020**

**Disclaimer: Due to the "take-home" nature of the Final Exam, this practice test does not necessarily look similar to the actual exam.**

**Q1**. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 7 \\ 1 & 1 & -3 \end{bmatrix}$

(a) Find the characteristic polynomial of $A$.

(b) Diagonalize $A$, if possible. If $A$ cannot be diagonalized, explain why.

**Q2**. Compute the determinant of the following matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & k & k^2 & k^3 \end{bmatrix}$. Your solution should be a polynomial in $k$.

(*Hint:* Recall that $k^2 - 1 = (k-1)(k+1)$ and $k^3 - 1 = (k-1)(k^2+k+1)$. )

**Q3**. Let $V$ be the vector space of continuous functions that are linear combinations of $f(t) = \cos t$, $g(t) = t^2 - 1$, $h(t) = t^2 + 1$, and $r(t) = t^2 + \cos t$.

(a) Find a basis $\mathcal{B}$ for $V$.

(b) Find the $\mathcal{B}$-coordinates for $p(t) = 3\cos t + t^2$, where $\mathcal{B}$ is the basis from part (a).

**Q4**. Consider the matrix $A = \begin{bmatrix} 6 & 6 \\ -3 & 0 \end{bmatrix}$.

(a) Find the eigenvalues of $A$.

(b) Find an invertible matrix $P$ such that $A = PCP^{-1}$ for some rotation-scaling matrix $C$ (i.e., $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ for some real numbers $a, b$).

**Q5**. It is known that the matrix $B = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ is invertible. Suppose $C$ is a $3 \times 3$ matrix such that $BCB = I_3$ (the $3 \times 3$ identity matrix). Is $C$ necessarily invertible? If yes, find the inverse of $C$. If no, explain why.

**Q6**. Let $\mathbb{R}^{2\times 2}$ denote the vector space of $2 \times 2$ real matrices, and define a transformation

$$S : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}\,,$$
$$S(A) = A^T - A\,,$$

where $A^T$ is the transpose of the matrix $A$.

(a) Show that the transformation $S$ is linear.

(b) Find a basis of the range of $S$ and a basis of the null space of $S$. Explain how the dimensions of these subspaces are related.

**Q7**. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 2 & 0 \\ 5 & 2 & 2 \\ -2 & -2 & 1 \end{bmatrix}$. Find

(a) a basis of $Nul(A)$,

(b) a basis of $Col(A)$,

(c) a basis of $Row(A)$,

(d) the rank of $A$.