You may bring one 8.5" x 11" sheet of **notes** (both sides) to all exams.

You may bring a calculator for the exam,

MATH 471 - PRACTICE EXAM 1

Tom Weston's web page

Problem 1. Give a careful proof of the following statement: for $a, b, c \in \mathbf{Z}$, if $a|b$ and $b|c$, then $a|c$.

Problem 2. Determine whether or not each of the following equations has a solution $x, y \in \mathbf{Z}$. Explain your answers. (You need not exhibit solutions when they exist.)

(a) $176x + 143y = 17$
(b) $176x + 143y = -44$
(c) $101x + 47y = 19$

Problem 3. Find all solutions to the linear equation

$$7x + 9y = 120$$

with $x, y \in \mathbf{Z}^{+}$.

Problem 4. Find all positive integers n such that $12|n$ and $n|816$.

Problem 5. Find all solutions to

$$11x + 17y = 305$$

with $x, y \in \mathbf{Z}^{+}$.

Problem 6. EXTENDED

(a) Use Euclid's algorithm to find the greatest common divisor of 222 and 189.
(b) Find $x, y \in \mathbf{Z}$ such that $73x + 50y = 1$.

Problem 7. Prove that for integers a, b, c, if $a \mid c$ and $b \mid c$, then $\operatorname{lcm}(a, b) \mid c$. (You may use any results from class in your proof.)

Problem 8. a) Find integers x, y such that $17x + 31y = 1$.
b) Use Euclid's algorithm to determine the greatest common divisor of 2121 and 1407.

Problem 9. Let n be a positive integer and let m be a divisor of n. Give a formula in terms of the prime factorizations of n and m for the number of positive integers d such that $d \mid n$ and $m \mid d$.

not covered (ch 13) → **Problem 10.** Factor $6 + 12i$ into primes in $\mathbf{Z}[i]$.

Problem 11.

(a) Use Euclid's algorithm to find the greatest common divisor of 91 and 35.
(b) Find $x, y \in \mathbf{Z}$ such that $91x + 35y = 7$.
(c) Find $x, y \in \mathbf{Z}$ such that $91x + 35y = 7$ and y is positive with one's digit 3.

Problem 12. Let a, b, q, r be integers such that $a = bq + r$. Prove that $(a, b) = (b, r)$.

Homework 5/ Practice Midterm 1

LIUBOMIR CHIRIAC

1. (a) Show that any integer can be written in the form $44X + 17Y$ where X and Y are integers.
   (b) Find integers x and y such that $44x + 17y = 100$.
2. Solve the system of congruences
$$x \equiv 1 \pmod 3,$$
$$x \equiv 2 \pmod 5,$$
$$x \equiv 3 \pmod 7.$$
   (Hint: Use the first two congruences to find a single congruence $\pmod{15}$. Then combine this with the last congruence.)
3. (a) A common error in banking is to interchange two of the digits in an amount. Prove that the difference between the correct amount and the amount with the two digits interchanged is always divisible by 9.
   (b) A palindrome is a number that reads the same backward and forward, e.g. 1991, 23577532. Prove that a palindrome with an even number of digits is always divisble by 11.
4. Let a, b, c be positive integers.
   (a) Prove that $\gcd(a, b, c) = \gcd(\gcd(a, b), c)$.
   (b) Prove that the Diophantine equation
$$aX + bY + cZ = 1$$
   has a solution if and only if $\gcd(a, b, c) = 1$. (Hint: Use (a) to reduce the problem to two variables.)
5. (a) How many positive integers less than 101 have an *odd* number of positive divisors?
   (b) How many positive integers k have the property that
$$\operatorname{lcm}(6^6, 8^8, k) = 12^{12}?$$

Remark. For Problems 4 and 5 you may use (without proof) the following fact: if
$$m = p_1^{a_1} \dots p_r^{a_r}, n = p_1^{b_1} \dots p_r^{b_r}, \text{ and } k = p_1^{c_1} \dots p_r^{c_r},$$
then
$$\gcd(m, n, k) = p_1^{d_1} \dots p_r^{d_r} \text{ and } \operatorname{lcm}(m, n, k) = p_1^{e_1} \dots p_r^{e_r},$$
where $d_i = \min(a_i, b_i, c_i)$ and $e_i = \max(a_i, b_i, c_i)$.