Assignment 4 (Due Tuesday March 1).

Section 30 page 94: 9, 10 (Method 1: check that its first and second partials are continuous and satisfies the Laplace equation. Method 2: realize this function as the real part of an analytic function).

Section 32 page 99: 1, 2 (c), 3, 5, 6

Additional Problem: Let $\arctan : \mathbb{R} \rightarrow \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$ be the inverse of the function $\tan : \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \rightarrow \mathbb{R}$. Let $U := \{ z = x + iy : x > 0 \}$ be the open right half-plane. Set

$$f(z) := \ln(x^2 + y^2) + i\arctan\left(\frac{y}{x}\right),$$

for $z = x + iy$ in $U$.

1. Show that $f(z) = \Log(z)$, for all $z \in U$, where $\Log(z)$ is the principal branch of $\log(z)$. Conclude that $f(z)$ is analytic.

2. Provide a second proof that $f(z)$ is analytic by checking that the assumptions of the Theorem page 63 in section 21 are satisfied (the partials of the real and imaginary components are continuous, and the Cauchy-Riemann equations are satisfied).

3. Use the function $\arccos : (-1, 1) \rightarrow (0, \pi)$ in order to express the imaginary part of $\Log(z)$ in the upper-half-plane $\mathbb{H} := \{ z = x + iy : y > 0 \}$. 