1. (36 points) Let \( z = \frac{6}{\sqrt{2} - \sqrt{2}i} \). Compute the following (in cartesian or polar form):

   a) The polar form of \( z \).
   
   b) \( |z^3| \)
   
   c) \( \log(z^6) \)
   
   d) All values of \( z^{1/2} \). How many different values are there?
   
   e) All values of \( z^i \). How many different values are there?

2. (10 points) Let \( f(z) \) be an entire function satisfying \( |f(z)|^2 = 2 \) for all \( z \). Prove that \( f \) must be a constant function. **Hint:** Show that the conjugate function \( \overline{f(z)} \) must be entire. Then use the Cauchy-Riemann equations to prove that \( f'(z) = 0 \).

3. (18 points) a) Compute the Cartesian coordinates of \( \sin(2i) \).
   
   b) Find the set of points in the plane, where the function \( \frac{z}{\sin(z) - 2i \cos(z)} \) is differentiable. Justify your answer!

4. (18 points) a) Prove that the function
   \[
   u(x, y) = e^x \sin(y) + e^y \cos(x) + 2xy
   \]
   is harmonic on the whole of \( \mathbb{R}^2 \).
   
   b) Find a harmonic conjugate \( v \) of the function \( u \).
   
   c) Find an entire function \( f(z) \) such that \( \text{Re}(f) = u \). Your answer must be expressed as a function of \( z = x + iy \), not \( x \) and \( y \).

5. (18 points) a) Find the image of the horizontal line \( y = 1/4 \) under the function \( f(z) = e^{\pi z} \).
   
   b) Find the image, under the principal branch of \( \log(z) \), of the set
   \[
   \{ z \text{ such that } |z| < 1 \text{ and } \text{Re}(z) > 0 \}
   \]
   (the right half of the unit disk).