MATH 300 MIDTERM REVIEW, FALL 2015

Chapter 1:

1. Determine which of the following sentences are statements. What are the truth values of those that are statements.
(a) $\sqrt{2}$ is not a rational number.
(b) Is $\sqrt{2}$ a rational number ?
(c) Show that $\sqrt{2}$ is not a rational number.
(d) If $1 = 0$, then $5 = 4$.
2. Write the truth tables for each expressions.
(a) NOT (P OR Q)
(b) (P AND Q) implies R
(c) (NOT P) implies (Q is equiv to R).
3. Is (P implies Q) equivalent to (NOT P implies NOT Q)? Give reasons.
4. Negate each expression, and simplify your answer.
(a) $\forall x \ \forall y, (x \geq y)$; (b) $\exists y \ \forall x, (x^2 + y^2 = 1)$.
5. Determine whether the following are equivalent, and explain why.
(a) $\exists x, (P(x) \text{ OR } Q(x))$ $(\exists x, P(x)) \text{ OR } (\exists x, Q(x))$;
(b) $\exists x, (P(x) \Rightarrow Q(x))$ $(\exists x, P(x)) \Rightarrow (\exists x, Q(x))$.
6. Let A, B be two sets. Show that $A \cap B = A \cup B$ if and only if $A = B$ (by direct proof method).
7. Show that if $x^2 + 2x^3 = 5$, then $x < 2$ (by contrapositive method).
8. Show that the equation $x^2 + x + 1 = 0$ has no real solutions (by contradiction).
9. Show that the statement "if $x^2 > 4$, then $x > 2$" is false (by counterexample).
10. Prove the following statements:
(a) If $m^2 + n^2$ is an odd integer, then m is odd or n is odd;
(b) There are no integer solutions to $2x^3 + 2x + 3 = 0$.
11. Disprove: $\forall x \in \mathbb{R}$, if $x^3 + 100x - 100 > 0$, then $x > 1$.

Chapter 4:

12. Prove using Mathematical Induction: $\forall n \in \mathbb{P}$,

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \geq 2 - \frac{1}{n}.$$

13. Prove using Mathematical Induction: Let $x \neq -1$. $\forall n \in \mathbb{P}$,

$$1 - x + x^2 + \cdots + (-1)^n x^n = \frac{1 - (-x)^{n+1}}{1 + x}.$$

14. A sequence of integers $x_1, x_2, x_3, \cdots$, is defined by $x_1 = 1, x_2 = 5$ and the recursion

$$x_n = 5x_{n-1} - 6x_{n-2}, \forall n \geq 3.$$

Find an expression for x_n and use Mathematical Induction to prove that the expression is correct.

15. Find an expression for

$$S_n = 1 - 3 + 5 - 7 + \cdots (-1)^{n+1}(2n-1), \text{ where } n = 1, 2, 3, \cdots,$$

and prove the expression for S_n is correct.

Chapter 2:

Use the Division or Euclidean Algorithm in the following problems.

16. Find the quotient and remainder when b is divided by a.
(a) $a = 2011$, $b = 231$, (b) $a = -17$, $b = 182$.
17. Find $gcd(a, b)$ and $lcm(a, b)$.
(a) $a = 598$, $b = 132$, (b) $a = 432$, $b = -282$.

Use the Extended Euclidean Algorithm in the following problems.

18. Write $gcd(100, 35)$ in the form $100x + 35y$ for some $x, y \in \mathbb{Z}$.
19. Find one integer solution, if there is one, of the linear Diophantine equations:
(a) $14x + 18y = 9$, (b) $11x + 15y = 5$.
20. Find all integer solutions of the linear Diophantine equation $15x - 24y = 9$.
21. Find all positive integer solutions of the linear Diophantine equation $20x + 25y = 200$.

Miscellaneous.

22. Show that if a is odd and $a \mid 2b$, then $a \mid b$.
23. Determine whether 191 is prime or not.