35 (page 83): $29x \equiv 43 \pmod{128}$

$\gcd(29, 128) = 1$, so solutions exist.
We find a particular solution by solving the Diophantine Equation

$$29x + 128y = 43.$$

First use the Extended Eucleadian Algorithm to solve $29x + 128y = 1$

$$128y_i + 29x_i = r_i$$

y_i	x_i	r_i	q_i
1	0	128	
0	1	29	
1	-4	12	4
-2	9	5	2
5	-22	2	2
-12	53	$\boxed{1}$	2

gcd

$29 \cdot 53 - 12 \cdot 128 = 1$. Multiply both sides by 43

$$29 \cdot (\underbrace{43 \cdot 53}_{2279 \equiv 103 \pmod{128}}) - (12 \cdot 43) \cdot 128 = 43.$$

So $x_0 = 2279 \equiv 103 \pmod{128}$.

The general solution is $x \equiv 103 \pmod{128}$ (unique congruence class, by Theorem 3.54, since $\gcd(29, 128) = 1$).

44) Find the inverse of [23] in $\mathbb{Z}_{41}$.

Answer: Solve

$$23x \equiv 1 \pmod{41}$$

$$\Leftrightarrow$$

$$23x + 41y = 1 \quad \text{for some } y \in \mathbb{Z}$$

Extended Euclidean Alg

$$41 y_i + 23 x_i = r_i$$

y_i	x_i	r_i	q_i
1	0	41	
0	1	23	
1	-1	18	1
-1	2	5	1
4	-7	3	3
-5	9	2	1
9	-16	$\boxed{1}$ gcd	1

So $\quad 23(-16) + 41 \cdot 9 = 1$

$-16 \equiv 25 \pmod{41}$, and so

[25] is the inverse of [23] in $\mathbb{Z}_{41}$

47) $([x]-[2])([x]-3) = [0]$ in $\mathbb{Z}_6$.

$$[x^2] - \underbrace{[5x]}_{\equiv \underbrace{[5]}_{\equiv -1}[x]} + [6] = [0] \quad \text{in } \mathbb{Z}_6$$

$$\underbrace{[x^2 + x]}_{\text{mod } 6} \equiv 0 \quad \text{in } \mathbb{Z}_6.$$

x	0	1	2	3	4	5
x^2+x	0	2	0	0	2	0

So $[x] = [0], [2], [3]$, or $[5]$.

48) For what a does $x^2 \equiv a \pmod 7$ have a solution?

Mod 7

x	0	1	2	3	4	5	6
x^2	0	1	4	2	2	4	1

If $a \equiv 1$ or 2 or $4 \pmod 7$, then the quadratic congruence has a solution. Otherwise, it does not.