Exercise Set 2 Page 50

51) $x =$ number of quaters
$y =$ " " dimes

$$25x + 10y = 95$$

One particular solution

$x_0 = 1, \quad y_0 = 7$

$\gcd(\underbrace{25}_{5^2}, \underbrace{10}_{2\cdot 5}) = 5$

General integral solution:

$$x = \underbrace{\underbrace{x_0}_{1} + n\frac{10}{5}}_{1+2n}, \quad y = y_0 - n\frac{25}{5} = 7 - 5n$$

$$\left.\begin{aligned} 0 \le x = 1 + 2n &\Rightarrow n \ge 0 \\ 0 \le y = 7 - 5n &\Rightarrow n \le \frac{7}{5} \end{aligned}\right\}$$

So, $n = 0$ and $n = 1$ are the only sol'n of.

Only two solution

$(x_0, y_0) = (1, 7)$, when $n = 0$

$(x_1, y_1) = (3, 2)$ when $n = 1$.

68) $6696 = 2^3 \cdot 3^3 \cdot \underbrace{31}_{\text{prime}}^{1}$

Every positive divisor d of 6696 has the form $d = 2^i \cdot 3^j \cdot 31^k$, where

$\underbrace{0 \le i \le 3}_{4\text{-possibilities}}$, $\underbrace{0 \le j \le 3}_{4\text{-possibilities}}$, $\overbrace{0 \le k \le 1}^{\text{two possibilities}}$.

Hence, 6696 has $4 \cdot 4 \cdot 2 = 32$ positive divisors.

69) $\text{lcm}(\underbrace{132}_{2^2 \cdot 3 \cdot 11}, \underbrace{9}_{3^2}) = 2^2 \cdot 3^2 \cdot 11 = 396$

70) $40 = 2^3 \cdot 5$

$144 = 2^4 \cdot 3^2$

$\gcd(2^3 \cdot 5, 2^4 \cdot 3^2) = 2^{\min\{3,4\}} \cdot 3^{\min\{0,2\}} \cdot 5^{\min\{1,0\}} =$

$= 2^3 = 8$

$\text{lcm}(40, 144) = 2^{\max\{3,4\}} \cdot 3^{\max\{0,2\}} \cdot 5^{\max\{1,0\}} =$

$= 2^4 \cdot 3^2 \cdot 5 = 720$