

An extra problem for section 6.6 on the cardinality of power sets

Given a set S , denote by $\mathcal{P}(S)$ the set of all subsets of S .

1. Let A and B be two disjoint sets. Construct a bijection between $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \times \mathcal{P}(B)$. Conclude the equality $\#(\mathcal{P}(A \cup B)) = \#(\mathcal{P}(A)) \cdot \#(\mathcal{P}(B))$, where the right hand side is defined in Section 6, Problem 102.
2. Prove the equality $\#(\mathcal{P}(\mathbb{N})) = \#(\mathcal{P}(\mathbb{N}) \times \mathcal{P}(\mathbb{N}))$, where \mathbb{N} is the set of positive integers. Hint: See problem 103.