# Extra problems on permutations

1. Use the composition table for $\mathcal{S}_3$ on page 152 in the text in order to show that every permutation in $\mathcal{S}_3$ can be written as a composition of $\sigma_4$ and $\sigma_5$ (the latter two are transpositions, see Problem 115 in Problem set 6). In other words, show that the set $\{\sigma_4, \sigma_5, \sigma_4 \circ \sigma_4, \sigma_4 \circ \sigma_5, \sigma_5 \circ \sigma_4, \sigma_4 \circ \sigma_5 \circ \sigma_4\}$ consists of all 6 permutations in $\mathcal{S}_3$.

2. Let $\sigma$ be the permutation of $\mathbb{P}_4 = \{1, 2, 3, 4\}$ given by

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ \sigma(1) & \sigma(2) & \sigma(3) & \sigma(4) \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$$

a) Exhibit $\sigma$ as a composition of transpositions.

b) Suppose that $\sigma = \tau_1 \circ \tau_2 \circ \cdots \circ \tau_k$, where $\tau_j$, $1 \leq j \leq k$, are $k$ transpositions. What can you say about the positive integer $k$. Justify your answer!