Problem set 6 page 157 #72:

$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, \quad \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$(\rho\circ\sigma)(1) = \rho(\sigma(1)) = \rho(2) = 1$

$(\rho\circ\sigma)(2) = \rho(3) = 4$

$(\rho\circ\sigma)(3) = \rho(4) = 3$

$(\rho\circ\sigma)(4) = \rho(1) = 2$

$$\rho\circ\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$$

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$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$ satisfies $\rho^2 = 1$ (identity)

So $\rho^{-1} = \rho$.

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$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$$

$$\tau^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$$

Problem set 6 page 157 #80:

Let S be a finite set of cardinality $n \geq 3$. We construct permutations σ, τ of S, such that $\sigma \circ \tau \neq \tau \circ \sigma$.

Let x, y, and z be three distinct elements of S. Let σ be the transposition interchanging x and y (and fixing all other elements) ($\sigma = (x, y)$).

Let τ be the transposition interchanging y and z (and fixing all other elements. Then

$$(\sigma \circ \tau)(x) = \sigma(\tau(x)) = \sigma(x) = y.$$

$$(\tau \circ \sigma)(x) = \tau(\sigma(x)) = \tau(y) = z.$$

Hence, $(\sigma \circ \tau)(x) \neq (\tau \circ \sigma)(x)$. Thus

$$\sigma \circ \tau \neq \tau \circ \sigma.$$

Q.E.D