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a) Let E be the set of even positive integers. Then $\#E = \#\underline{P}$.

Proof: Let $f : \underline{P} \to E$ be given by

$$f(n) = 2n.$$

Let $g : E \to \underline{P}$ be $g(n) = n/2$.

Then $(f \circ g)(n) = 2(n/2) = n$, so $f \circ g = 1_E$.

$(g \circ f)(n) = \frac{1}{2}(2n) = n$, so $g \circ f = 1_{\underline{P}}$

Hence, f is a bijection, by the Inversion Theorem 6.51. Thus $\#(E) = \#(\underline{P})$, by definition.

b) Let D be the set of odd positive integers. Then $\#D = \#\underline{P}$.

Proof: Let $f : \underline{P} \to D$ be $f(n) = 2n - 1$.

Let $g : D \to \underline{P}$ be $g(n) = \frac{n+1}{2}$.

Then $f \circ g = 1_D$, $g \circ f = 1_{\underline{P}}$ and so $\#D = \#\underline{P}$, by the argument in part a.