Math 300.1 Quiz 3 Fall 2017

Name: Solution

Let $X = \{x \in \mathbb{R} : x \geq -3\}$ and $Y := \{y \in \mathbb{R} : y \leq 1\}$. Prove that the function $f : X \to Y$, given by $f(x) = 1 - \sqrt{3+x}$ is bijective and find a formula for the inverse function. Hint: Use the following steps: 1) Find a candidate function $g : Y \to X$ for the inverse by setting $y = f(x)$ and solving for x as a function of y. 2) Check that g is indeed the inverse of f by computing the two compositions $f \circ g$ and $g \circ f$ and stating and using the definition of the inverse function. 3) Explain why it follows that f is bijective.

10 pts

1) $y = 1 - \sqrt{3+x}$

$1 - y = \sqrt{3+x}$

$(1-y)^2 = 3+x$

$x = (1-y)^2 - 3$. Let $g : Y \to X$ be given by

$$\boxed{g(y) = (1-y)^2 - 3,}$$

Note that $(1-y)^2 - 3 \geq -3$, for all $y \in \mathbb{R}$ and in particular for $y \in Y$. Hence g is well defined.

2) For $y \in Y$,

$(f \circ g)(y) = f(g(y)) = 1 - \sqrt{3 + g(y)} = 1 - \sqrt{3 + [(1-y)^2 - 3]} =$

$= 1 - \sqrt{(1-y)^2} = 1 - (\underbrace{1-y}_{\geq 0}) = y.$

$\leftarrow$ since $y \leq 1$

Hence $f \circ g = 1_Y$

Let $x \in X$. Then

$(g \circ f)(x) = g(f(x)) = (1 - f(x))^2 - 3 = (1 - \underbrace{[1 - \sqrt{3+x}]}_{\sqrt{3+x}})^2 - 3 =$

$= (3+x) - 3 = x.$

So $g \circ f = 1_X$. Hence, g is the inverse of f, by definition.

0 pts

3) A function f has an inverse, if and only if it is bijective, by the Inversion Theorem 6.51. Thus, f is bijective.