Math 300.1   Quiz 2   Fall 2017

Name: **Solution**

1. (50 points) What is the remainder, when 2^{86} is divided by 9. Justify your answer.

$2^3 = 8 \equiv -1 \pmod 9$.

So $2^{86} = 2^{84} \cdot 2^2 = (2^3)^{28} \cdot 4 \equiv (-1)^{28} \cdot 4 \equiv 4 \pmod 9$

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$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (\bmod\ 9)$

Hence, the remainder is 4.

2. (50 points) Is $14^{111} + 17^8$ divisible by 15? Justify your answer.
   Hint: $2^4 \equiv 1 \pmod{15}$.

$14 \equiv -1 \pmod{15}$. Hence $14^{111} \equiv (-1)^{111} \equiv -1 \pmod{15}$

$17^8 \equiv 2^8 = (2^4)^2 \equiv 1^2 \equiv 1 \pmod{15}$.

Hence, $14^{111} + 17^8 \equiv -1 + 1 \equiv 0 \pmod{15}$

Thus, the sum is divisible by 15.