Exatra problem on projections

Let W be the plane in $\mathbb{R}^3$ spanned by $u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $u_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

1. Find the projection $Proj_W(b)$ of $b = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ to W.

2. Verify that your answer in part 1 satisfies the definition of $Proj_W(b)$, i.e., show that $b - Proj_W(b)$ is orthogonal to W.

3. Find the distance from b to W.

4. Find a least square solution of the equation $Ax = b$, where $A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix}$ is the 3×2 matrix with columns u_1 and u_2. I.e., find a vector x in $\mathbb{R}^2$ which minimizes the length $\|Ax - b\|$. Hint: Solve $Ax = Proj_W(b)$.

5. Find the coefficients c_0, c_1 of the line $y(x) = c_0 + c_1 x$ which best fits the three points $(x_1, y_1) = (1, 2)$, $(x_2, y_2) = (-2, 1)$, $(x_3, y_3) = (1, -2)$ in the x, y plane. The line should minimize the sum

$$\sum_{i=1}^{3} [c_0 + c_1 x_i - y_i]^2. \tag{1}$$

Justify your answer!
Hint: Set $\vec{c} := \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}$. Show that the sum in equation (1) is the square of the distance from $A\vec{c}$ to b, where A is the matrix in part 4. Next explain why the solutions to parts 4 and 5 are the same vector in $\mathbb{R}^2$..