1. a: Write the system of equations below as a matrix equation.
   b: Find all solutions using Gauss elimination:

$$-x - 2y - 4z = 0, -x + 3y + z = -5, 2x + y + 5z = 3.$$

2. What does it mean for a vector to be in the image of a matrix A.
   Let A be the matrix $\begin{pmatrix} 1 & 2 & 5 \\ -2 & 0 & -2 \\ 3 & -1 & 1 \end{pmatrix}$. Is $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ an element of the image of A?
   Why?

3. Define what it means for a set s to be a basis of a subspace $V \subset \mathbb{R}^n$. Let

$$A = \begin{pmatrix} 1 & 2 & 3 & -1 \\ -1 & 0 & 1 & -1 \\ -1 & 4 & 3 & -5 \end{pmatrix}.$$

   Give a set of vectors that span $im(A)$ and that are independent.

4. Let A be a n by m matrix, so A gives a linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$. Let $x_1, x_2 \in \mathbb{R}^m$. Assume that $A(x_1) = 0$ and that $A(x_2) = b$.Explain why $A(x_1+x_2) = b$

5. Let A be a two by two matrix that rotates by angle $2\pi/6$. a: Find A.
   b: Give a geometric explanation why $A^6 = I_2$. Here I_2 denotes the 2 by 2 identity matrix.

6. Solve the equation

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 0 & 2 \\ 2 & 1 & -1 \end{pmatrix} x = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}.$$

   for $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ by finding the inverse of the given matrix.

7. Compute the product AB of the two matrices A, B given below, if possible. If it is not possible say why it is not possible.

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & -2 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & 0 & 0 \\ 4 & 8 & 7 \end{pmatrix}$$

8. Let W denote the set of vectors orthogonal to the vector $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$. Find a basis of W.

9. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 1 & 1 & 3 \\ -2 & 1 & -3 \end{pmatrix}.$$

Let $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$. Find equations in b_1, b_2, b_3, b_4 so that the equation $Ax = b$ can be solved. Find a basis of the image of A.

10. True or False. It is necessary to give an explanation.
a: The image of a 3 by 4 matrix A is a subspace of $\mathbb{R}^4$.
b: If the kernal of a matrix A consists of the zero vector only, then the column vectors of A must be linearly independent.
c: If u, v, w are in a subspace of R^n, then so is $3u - v - w$.
d: The function

$$T : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x - y \\ x + y \end{pmatrix}$$

is a linear transformation.
e: If A is an invertible $n \times n$ matrix, then the reduced row ecelon form of A is I_n.

f: The formula $AB = BA$ holds for all $n \times n$ matrices A and B.
g: If the 4×4 matrix A has rank 4, then any linear system with coefficient matrix A will have a unique solution.