1. (20 points) Let \( \vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \) and let \( \vec{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \). Let \( V \) be the subspace spanned by \( \vec{v}_1 \) and \( \vec{v}_2 \).

a. (5 pts) Prove that \( \vec{v}_1 \) is not perpendicular to \( \vec{v}_2 \).

b. (8 pts) Find an orthonormal basis for \( V \).

c. (7 pts) What is the matrix for orthogonal projection onto \( V \)?

2. (17 points) Find the quadratic polynomial \( p(t) = a + bt + ct^2 \) that best (in the least squares sense) fits the following data.

<table>
<thead>
<tr>
<th>( t )</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(1)</td>
<td>(1.5)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

3. (28 points) Let \( V \subseteq C^\infty \) be subspace spanned by \( \{e^x, xe^x, x^2e^x\} \). Let \( B \) be the ordered basis \( B = (e^x, xe^x, x^2e^x) \).

a. (4 pts) What is the dimension of \( V \)?

b. (8 pts) Let \( D : V \rightarrow V \) be the linear transformation given by \( D(f) = f' \). Express \( D \) as a matrix with respect to the basis \( B \). i.e. Compute \( \text{Mat}_B^B(D) \).

c. (8 pts) Let \( A = \text{Mat}_B^B(D) \). You can check that:

\[ A^3 - 3A^2 + 3A - 1 = 0. \]

Consider the function \( f(x) = 2e^x - 13xe^x + \sqrt{2}x^2e^x \). What does the above tell you about \( f''' - 3f'' + 3f' - f \)?

d. (8 pts) Suppose you want to find functions \( u \) such that

\[ u'''(x) - 3u''(x) + 3u'(x) - u(x) = x. \]

Verify that \( u(x) = -x - 3 \) is a solution. Find another one.

4. (15 points) Find a basis for the space perpendicular to the solutions of

\[ x_1 + 3x_2 - x_3 + x_4 = 0 \]
\[-2x_1 + 2x_2 + x_3 + x_4 = 0\]

5. (20 points) Let \( P_5 \) denote the vector space of polynomials of degree at most 5. Let \( S \subseteq P_5 \) denote the subset of polynomials \( p \) such that

\[ p''(2) = p(4). \]

Show that \( S \) is a subspace of \( P_5 \) and compute a basis of \( S \).
Before test 2:

1. Make sure you can define the following words:
   
   (a) linear transformation
   (b) subspace
   (c) linearly independent
   (d) rank
   (e) kernel
   (f) image
   (g) span
   (h) dimension
   (i) similar matrices
   (j) vector space
   (k) transpose of a matrix
   (l) orthogonal matrix
   (m) symmetric matrix
   (n) skew-symmetric matrix
   (o) orthonormal basis

2. Make sure you can do Gaussian Elimination and Gram-Schmidt, and you know what each is good for.

3. Make sure you can solve a linear system.

4. Make sure you can state the Rank-Nullity Theorem and fully appreciate all of its consequences.