1. Write the system of equations as a matrix equation and find all solutions using Gauss elimination:

\[ \begin{align*}
    x + 2y + 4z &= 0, \\
    -x + 3y + z &= -5, \\
    2x + y + 5z &= 3.
\end{align*} \]

2. What does it mean for a vector to be in the kernal of a matrix \( A \). Let \( A \) be the matrix

\[
\begin{pmatrix}
    1 & 2 & 5 \\
    -2 & 0 & -2 \\
    3 & -1 & 1
\end{pmatrix},
\]

Is \[
\begin{pmatrix}
    1 \\
    2 \\
    1
\end{pmatrix}
\]
an element of the kernal of \( A \)? Why?

3. Define what it means for a set \( s \) to be a basis of a subspace \( V \subset \mathbb{R}^n \). Let

\[
A = \begin{pmatrix}
    1 & 2 & 3 & -1 \\
    -1 & 0 & 1 & -1 \\
    -1 & 4 & 3 & -5
\end{pmatrix}.
\]

Give a set of vectors that span \( \text{ker}(A) \) and that are independent.

4. Let \( A \) be a \( n \) by \( m \) matrix, so \( A \) gives a function from \( \mathbb{R}^m \) to \( \mathbb{R}^n \). Let \( x_1, x_2 \in \mathbb{R}^m \). Assume that \( A(x_1) = A(x_2) \). Show that \( x_1 - x_2 \) is in the kernal of \( A \).

5. Let \( u = (u_1, u_2) \) be a vector of length 1. Let \( A \) be a matrix whose effect on the plane is to reflect about the line through the origin and \( u \). Let \( v = (-u_2, u_1) \). In terms of \( u \) and \( v \) what is \( A(u) \)? what is \( A(v) \)? Write \( e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) as a linear combination of \( u \) and \( v \). Use the answer to the previous question to compute \( A(e_1) \).

6. Solve the equation

\[
\begin{pmatrix}
    1 & 0 & -1 \\
    0 & 1 & 2 \\
    2 & 1 & -1
\end{pmatrix} \begin{pmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{pmatrix} = \begin{pmatrix}
    1 \\
    0 \\
    -1
\end{pmatrix},
\]

for \( x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \) by find the inverse of the given matrix.

7. Compute the product \( AB \) of the two matrices \( A, B \) given below, if possible. If it is not possible say why it is not possible.

\[
A = \begin{pmatrix}
    1 & 2 \\
    -1 & 0 \\
    3 & -2
\end{pmatrix},
\]

\[
B = \begin{pmatrix}
    -1 & 0 \\
    4 & 8
\end{pmatrix}
\]

The product matrix \( AB \) gives a function. What is the domain and what is the range of that function?
8. Find a basis of the subspace of $\mathbb{R}^3$ defined by $3x - y + z = 0$. What is the dimension of this subspace?

9. Consider the matrix

$$A = \begin{pmatrix}
1 & 0 & 2 \\
-1 & 2 & 0 \\
1 & 1 & 3 \\
-2 & 1 & -3
\end{pmatrix}$$

Let $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$. Find conditions on $b$ so that the equation $Ax = b$ can be solved. Find a basis of the image of $A$.

10. Let $V, W$ be subspaces of $\mathbb{R}^n$. Assume that $V \subset W$ and that the dimension of $V$ is equal to the dimension of $W$. Show $V = W$. 