Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ given by multiplication by a 3×3 matrix A. We will see in Chapter 5 that the linear transformation is a rotation of $\mathbb{R}^3$ about some line through the origin (the axis of rotation), if and only if A satisfies the following two conditions:
(i) $A^T A = I$ (in other words, the transpose of A is equal to the inverse of A) and
(ii) $\det(A) = 1$.

1. Show that the following two are matrices of rotations (check conditions (i) and (ii) above):

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

2. Consider a rotation given by some matrix A. Explain why any vector in the axis line of the rotation must be an eigenvector of A with eigenvalue 1.

3. Find a non-zero vector spanning the axis line of each of the two rotations given by the matrices A and B above. *Note: For the axis line of B, you will probably need to use the identity* $(\sqrt{2}+1)(\sqrt{2}-1) = 1$.