

TEST 1

Math 235
Fall 2006

Name: _____

Read all of the following information before starting the exam:

- Do all work to be graded in the space provided. If you need extra space, use the reverse of the page and indicate on the front that you have continued the work on the back.
- *You must support your answers with necessary work. My favorite number is three. Unsupported answers will receive zero credit.*
- Circle or otherwise indicate your final answers.
- You do not need to simplify expressions such as $\sqrt{11}$, e^2 , π with decimal approximations.
- This test has 9 problems and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- If you have read of all these instructions, check this box for one bonus point .
- Good luck!

1. (2 points) Go back and read the instructions on page one. What is my favorite number?

2. (12 points) Solve the system of linear equations:

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 4 \\ 0 & 2 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

3. (10 points) Let C be a 5×7 matrix (5 rows and 7 columns). Place a check in the correct box identifying each object.

Object	Subspace of \mathbb{R}^5	Subspace of \mathbb{R}^7	Not a subspace
Solutions to $C\vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$			
Solutions to $C\vec{x} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 4 \\ 3 \end{pmatrix}$			
$\ker(C)$			
$\text{im}(C)$			
My dog Duey			

4. (14 points) Suppose A is a square $n \times n$ matrix, and A similar to B . Write True or False for the following statements.

- a. (2 pts) _____ B is a $n \times n$ matrix.
- b. (2 pts) _____ There exists an invertible $n \times n$ matrix S such that $B = S^{-1}AS$.
- c. (2 pts) _____ B is similar to A .
- d. (2 pts) _____ If $A^3 + 2A + I$ is the zero matrix, then $B^3 + 2B + I$ is the zero matrix.
- e. (2 pts) _____ A^4 is similar to B^4 .
- f. (2 pts) _____ If $\text{rank}(A) = n$, then B is invertible.
- g. (2 pts) _____ If $\dim(\ker(A)) = 5$, then $\dim(\ker(B)) = 5$.

5. (14 points) Consider the subspace $V \subseteq \mathbb{R}^3$ of vectors perpendicular to $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

- a. (4 pts) Find a matrix A such that $V = \ker(A)$.

- b. (10 pts) Let \mathcal{B} be the ordered basis

$$\mathcal{B} = \left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right),$$

and let T be the linear transformation of projection to V . Compute $\text{Mat}_{\mathcal{B}}^{\mathcal{B}}(T)$.

6. (12 points) Let $T := \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the function defined by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ 2x_2 + x_3 \\ x_1 + x_3 \end{pmatrix}.$$

a. (6 pts) Express T as a matrix A .

b. (6 pts) Compute the dimension of the image of T .

7. (10 points) Let $T : \mathbb{R}^{12} \rightarrow \mathbb{R}$ be a linear transformation.

a. (5 pts) What are the possible dimensions of $\text{im}(T)$? Justify.

b. (5 pts) What are the possible dimensions of $\text{ker}(T)$? Justify.

- 8.** (14 points) Let $A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 1 \end{bmatrix}$. Then $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
- a.** (7 pts) Find a *basis* for the image of A .

- b.** (7 pts) Find a *basis* for the kernel of A .

- 9.** (12 points) Let $\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$. Then $\mathcal{B} = (\vec{u}, \vec{v})$ is an ordered basis of the subspace $V = \text{span } \mathcal{B} \subseteq \mathbb{R}^4$. Find the vector $\vec{w} \in V$ such that $[\vec{w}]_{\mathcal{B}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

SCRATCH PAPER

BONUS Survey: 1 bonus point total for completion

1. Suppose you are running in a race. You pass the second place runner. What place are you in now?
2. Out of 100 points, what do you think is your score on this test?
3. Out of 100 points, what do you think is the class average on this test?