Math 235 Midterm 1 Spring 2005

1. (20 points) a) Find the row **reduced** echelon augmented matrix of the system
$x_1 + x_2 + x_3 + x_4 = 4$
$x_2 - x_3 + 2x_4 + x_5 = 3$
$x_1 + 2x_2 + 5x_4 + x_5 = 9$

   b) Find the general solution for the system.

2. (18 points) Let $u_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} 3 \\ 2 \\ h \end{bmatrix}$, and $u_4 = \begin{bmatrix} h \\ 1 \\ 1 \end{bmatrix}$.

   Justify your answers to the following questions!

   a) For which real numbers h does the set $\{u_1, u_2, u_3, u_4\}$ span the whole of $\mathbb{R}^3$?

   b) For which values of h does the vector u_3 belong to the plane spanned by $\{u_1, u_2\}$

   c) For which values of h does the vector u_4 belong to the plane spanned by $\{u_1, u_2\}$?

   d) For which real numbers h is the set $\{u_1, u_2, u_3, u_4\}$ linearly independent?

3. (13 points) Set up a system of linear equations for finding the electrical **branch** currents $I_1, ..., I_6$ in the following circuit using i) the junction rule: the sum of currents entering a junction is equal to the sum of currents leaving the junction. ii) Ohm's rule: The drop in the voltage ΔV across a resistance R is related to the (directed) current I by the equation $\Delta V = IR$. iii) Kirchhof's circuit rule: the sum of the voltage drops due to resistances around any closed loop in the circuit equals the sum of the voltages induced by sources along the loop.

   Note: Do **not** *solve the system.*

   Not covered in Fall 2015 semester

4. (16 points) Determine if the statement is true or false. If it is true, give a reason. If it is false, provide a counter example. (credit will be given only if a valid justification is provided).

   (a) If A is a 4×3 matrix (4 rows and 3 columns), $\vec{b}$ is a vector in $\mathbb{R}^4$, and the equation $A\vec{x} = \vec{b}$ is consistent, then it has infinitely many solutions.

   (b) Let A be a square 3×3 matrix. If the equation $A\vec{x} = \vec{b}$ is consistent, for all vectors $\vec{b}$ in $\mathbb{R}^3$, then the columns of A are linearly independent.

   (c) Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$. For every three vectors v_1, v_2, v_3, in $\mathbb{R}^2$, the set $\{T(v_1), T(v_2), T(v_3)\}$ is linearly dependent (in $\mathbb{R}^3$).

   (d) Let A be a 3×4 matrix and b_1, b_2 two vectors in $\mathbb{R}^3$. If the vector equations $A\vec{x} = b_1$ and $A\vec{x} = b_2$ are both consistent, then so is the equation $A\vec{x} = b_1 - b_2$.

5. (15 points) a) Find two vectors v_1, v_2 in $\mathbb{R}^3$ which span the plane given by the equation

$$x_1 + 3x_2 - x_3 = 0.$$

b) Let v_1, v_2 be the two vectors from part a). Find the equation of the plane consisting of all vectors of the form $sv_1 + tv_2 + \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$, where s, t are real numbers.

6. (18 points) Find the standard matrix of each of the following linear transformations.

a) T is the map from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by

$T(x_1, x_2, x_3) = (2x_1 + x_2 - x_3, 5x_1 + 2x_2 + x_3, 9x_1 + 7x_2 - 5x_3)$.

b) T is the map from $\mathbb{R}^2$ to $\mathbb{R}^2$, which rotates points (about the origin) through $3\pi/4$ radians (counterclockwise).

c) T is the map from $\mathbb{R}^2$ to $\mathbb{R}^2$, which first reflects points through the vertical x_2 axis and then reflects points through the line $x_2 = x_1$.