Math 235 Midterm 1 Fall 2015

Your Name: ________________________________

Student ID: ________________________________

This is a 90 minutes exam. This exam paper consists of 5 questions. It has 9 pages.

The use of calculators is not allowed on this exam. You may use one letter size page of notes (both sides), but no books.

It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

1. (20 points) a) Find the row **reduced** echelon augmented matrix of the system
   $x_1 + 2x_2 + 3x_3 + 3x_4 + 5x_5 = 10$
   $x_1 + 3x_2 + 5x_3 + 4x_4 + 7x_5 = 14$
   $2x_1 + 5x_2 + 8x_3 + 7x_4 + 13x_5 = 25$

   b) Find the general solution for the system.

   $$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} =$$

   c) Find the general solution of the associated homogeneous system (replacing the constant, on the right hand side of the equations in part a, by zeros). Try to avoid computations. Justify your answer!

   $$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} =$$

   d) Find two vectors which span the solution set in Part c.

2. (20 points) Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} -1 \\ 2 \\ h \end{bmatrix}$.

   a) For which real numbers h is the vector $\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$ in span$\{v_1, v_2, v_3\}$? Justify your answer!

   b) For which values of h do the three vectors $\{v_1, v_2, v_3\}$ span the whole of $\mathbb{R}^3$? Justify your answer!

3. (20 points) For each of the following maps, determine if it is a linear transformation. If it is, find its standard matrix. If it is not, explain which property of linear transformations it violates (and why it violates it).

   a) T is the map from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by

   $T(x_1, x_2, x_3) = (x_1 + x_2 - x_3, x_1 + 2x_3, x_2 + 6x_3)$.

   b) T is the map from $\mathbb{R}^3$ to $\mathbb{R}^2$ defined by
   $T(x_1, x_2, x_3) = (x_1 + x_2, x_1 + 1 + 2x_3, 2x_1 - 3x_3)$.

   c) T is the map from $\mathbb{R}^2$ to $\mathbb{R}^2$, which sends each vector to its reflection with respect to the line $x_2 = -x_1$ (please, pay attention to the signs!).

4. (20 points)

   (a) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 4 \\ 1 & k \end{pmatrix}$. What value of k, if any, will make $AB = BA$? Justify your answer!

   (b) Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $R(\vec{x}) = A\vec{x}$ and $S : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $S(\vec{x}) = B\vec{x}$, where $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, given by $T(\vec{x}) = S(R(\vec{x}))$. Show all your work!

   (c) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,1} & b_{2,2} & b_{2,3} \end{pmatrix}$ an arbitrary 2×3 matrix. Explain why every column of the 3×3 matrix $C := AB$ must belong to the plane $x_1 - 2x_2 + x_3 = 0$ in $\mathbb{R}^3$.

5. (20 points) Determine if the statement is true or false. Justify your answer! (credit will be given only if a valid justification is provided).

   (a) If A is a 4×3 matrix, then there must be a vector $\vec{b}$ in $\mathbb{R}^4$, such that the equation $A\vec{x} = \vec{b}$ is inconsistent.

   (b) If the columns of a square $n \times n$ matrix are linearly independent, then they span the whole of $\mathbb{R}^n$.

   (c) There exist three vectors v_1, v_2, v_3 in $\mathbb{R}^5$, such that the set $\{v_1 + v_2, v_1 + v_3, 2v_1 + v_2 + v_3\}$ is linearly independent.