

1. (20 points) a) Find the row **reduced** echelon augmented matrix of the system

$$x_1 + x_2 + x_4 = 4$$

$$x_2 - x_3 + x_4 = 3$$

$$x_1 + x_3 + 2x_4 = 3$$

- b) Find the general solution for the system.

2. (16 points) Let $u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, and $u_3 = \begin{bmatrix} 1 \\ -1 \\ h \end{bmatrix}$.

Justify your answers to the following questions!

- a) For which real numbers h does the vector $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ belong to the span of $\{u_1, u_2, u_3\}$?

- b) For which real numbers h is the set $\{u_1, u_2, u_3\}$ linearly *dependent*?

3. (10 points) Set up a system of linear equations for finding the electrical **branch** currents I_1, \dots, I_6 in the following circuit using i) the junction rule: the sum of currents entering a junction is equal to the sum of currents leaving the junction. ii) Ohm's rule: The drop in the voltage ΔV across a resistance R is related to the (directed) current I by the equation $\Delta V = IR$. iii) Kirchhof's circuit rule: the sum of the voltage drops due to resistances around any closed loop in the circuit equals the sum of the voltages induced by sources along the loop.

*Note: Do **not** solve the system.*

Not covered in Fall 2015 semester.

4. (16 points) Determine if the statement is true or false. Justify your answer! (credit will be given only if a valid justification is provided).

(a) If A is a 4×3 matrix, then there must be a vector \vec{b} in \mathbb{R}^4 , such that the equation $A\vec{x} = \vec{b}$ is inconsistent.

(b) If the columns of a square $n \times n$ matrix are linearly independent, then they span the whole of \mathbb{R}^n .

(c) If v_1, v_2, v_3 is a linearly independent set of three vectors in \mathbb{R}^{15} , then the set $\{v_1 + v_2, v_1 + v_3, 2v_1 + v_2 + v_3\}$ is linearly independent as well.

(d) If T is a linear transformation from \mathbb{R}^n to \mathbb{R}^m , and $\{v_1, v_2, v_3\}$ is a linearly dependent set of three vectors in \mathbb{R}^n , then the set $\{T(v_1), T(v_2), T(v_3)\}$ is linearly dependent.

5. (8 points) Find two vectors v_1, v_2 in \mathbb{R}^3 which span the plane given by the equation

$$x_1 + 2x_2 - 3x_3 = 0.$$

Write down the general definition of the span of a set of vectors, and carefully explain why the above plane is equal to the span of the set you provided.

6. (15 points) Find the standard matrix of each of the following linear transformations. a) T is the map from \mathbb{R}^2 to \mathbb{R}^3 defined by

$$T(x_1, x_2) = (x_1 - x_2, 5x_1 + 2x_2, 9x_1 + 7x_2).$$

b) T is the map from \mathbb{R}^2 to \mathbb{R}^2 , which rotates points (about the origin) through $3\pi/4$ radians **counterclockwise**.

c) T is the map from \mathbb{R}^2 to \mathbb{R}^2 , which first reflects points through the line $x_2 = x_1$ and then reflects points through the horizontal x_1 axis. **Justify your answer!**

7. (15 points) Let $D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ a) Give an example of a 2×2 matrix A , such that $AD \neq DA$. Justify your answer.

b) Find the general form of a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, which satisfies $AD = DA$. Justify your answer. *Hint:* Treat the above equation as a system of linear equations in the variables a, b, c, d .