MATH 132H SPRING 2006 EXAM 2

1. (16 points) For each of the following improper integrals, determine if it converges or diverges. If convergent, evaluate the integral. Otherwise, explain why it diverges.

   a) $\displaystyle\int_0^\infty \frac{x}{1+x^2}dx$

   b) $\displaystyle\int_1^\infty \frac{\ln(x)}{x^2}dx$ Hint: use integration by parts.

2. (a) (12 points) Use trigonometric substitution to evaluate the following integral.

   $$\int \sqrt{1-9x^2}dx$$

   Many of you have calculators, which calculate the answer: $\dfrac{\arcsin(3x)}{6} + \dfrac{x\sqrt{1-9x^2}}{2} + C$. Credit will be given only for an answer using trigonometric substitution showing all your algebraic steps. You may use the identities $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ and $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

   (b) (3 points) Use part 2a to find the area enclosed by the ellipse $9x^2 + y^2 = 1$. Justify your answer!

3. (8 points) Use partial fractions to evaluate the following integral algebraically. Show all your steps.

   $\displaystyle\int \frac{1}{x^2-x}dx =$

4. (30 points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. Show all your work! Explain, in particular, which test you used and why the conditions of the test are satisfied.

   a) $\displaystyle\sum_{n=1}^\infty n\left(\frac{2}{3}\right)^n$

   b) $\displaystyle\sum_{n=1}^\infty (-1)^n \frac{n^2-4n}{2n^2+7n+5}$

   c) $\displaystyle\sum_{n=1}^\infty (-1)^n \frac{5n}{n^2+2n}$

   d) $\displaystyle\sum_{n=2}^\infty \frac{1}{n[\ln(n)]^2}$ Hint: Use the integral test.

5. (15 points) a) Find a power series representation for the function $f(x) = \dfrac{x}{1+3x^4}$.

   b) Determine the interval of convergence of the power series in part a).

   c) Find a power series representation of $\int \frac{x}{1+3x^4}$.

6. (16 points) a) Use the formula for the coefficient of the Taylor series, in order to determine the Taylor series for $f(x) = \sin(x)$ centered at $a = 0$ (i.e., the Maclaurin series). Show all your work! Credit will **not** be given for an answer without a justification.

   b) Recall Taylor's Remainder Inequality: If $\left|f^{(n+1)}(x)\right| \leq M$, for $|x-a| \leq d$, then the remainder $R_n(x)$, of the Taylor series centered at $x = a$, satisfies the inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}, \quad \text{for } |x-a| \leq d.$$

   Use Taylor's remainder inequality to determine the **minimal** degree n, of the Taylor polynomial $T_n(x)$ centered at 0, needed to approximate $\sin(0.1)$ to within 5 decimal digits. Justify your answer!