DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 132H SPRING 2006
EXAM 1 PART 2

Your Name: ________________________________

This exam paper consists of 5 questions. It has 7 pages.

On this exam, you may use a calculator and one side of one letter size page of notes, but no books.

It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

If you draw a graph, you must include the value of the range variables and show the tick marks on the axes, if any.

If your drawing includes the graphs of more than one function, you must label them so that we can tell which is which.

1. (8)	____________
2. (8)	____________
3. (8)	____________
4. (15)	____________
5. (15)	____________
6. (15)	____________
7. (15)	____________
8. (16)	____________
TOTAL (100)	

4 (15) Determine the following derivatives. Briefly justify each answer.

a) $\dfrac{\partial}{\partial x}\displaystyle\int_1^{x^2} \sin(t+t^2)dt.$

b) $\dfrac{\partial}{\partial x}\displaystyle\int_x^{10} \ln(1+t^2)dt$

c) $\dfrac{\partial}{\partial x}\displaystyle\int_0^{\pi} \sin(x)dx$

5 (15) a) Sketch the region bounded by the y-axis, the curve $y = \dfrac{12}{2+x^2}$, and the curve $y = x^2 - 2$.

b) Use the Cylindral-shells method to find the volume of the solid obtained by revolving about the y-axis the region in part a).

6 (15) a) Approximate the integral $\int_0^1 e^{(x^2)}dx$ by a Riemann sum that uses 5 equal-length sub-intervals and **left**-hand endpoints as sample points. (Show the individual terms of the Riemann sum before you calculate the value of the sum).

b) Interpret the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1-\left(\frac{i}{n}\right)^2} \cdot \left(\frac{1}{n}\right)$ as the area of a region. Justify this interpretation! Graph this region. Use your interpretation to evaluate the limit.

7 (15) The graph of the curve $x = (y^3 + 5y^2 + y)^{\frac{1}{3}}$, $y > 0$, is revolved about the y-axis to form the outer surface of a water container. Water is being poured in at a constant rate of 10 centimeters cubed per second. Assume the x and y units are in centimeters.

a) Use the disk (washer) method to set-up an integral for the volume $V(h)$ of the water in the container, when the water level reaches the horizontal line $y = h_{\text{cm}}$ (for some constant height h). Do **NOT** evaluate the integral.

b) Use the fact that $\frac{\partial V(h(t))}{\partial t} = 10_{\text{cm}^3/\text{sec}}$, the Fundamental Theorem of Calculus, and the Chain Rule, to show that the rate of change $\frac{\partial h}{\partial t}$, of the height $h(t)$ of the water in the container with respect to time, is equal to 10 divided by the horizontal-surface-area of the water

$$\frac{\partial h}{\partial t} = \frac{10}{\pi(h^3 + 5h^2 + h)^{2/3}} \text{ cm/sec.}$$

8 (16) A ball is thrown upward from a tower window, 200 feet above the ground, with initial velocity $v_0 = 48$ feet per second. Its acceleration, t seconds afterwards, is $v'(t) = -32$ ft/sec^2.

a) Find the velocity $v(t)$ of the ball t seconds after it is thrown, but before it hits the ground.

b) Find the height $h(t)$ of the ball above the ground t seconds after it is thrown.

c) Find the total distance traveled by the ball during the time interval $0 \leq t \leq 2$ seconds.