REVIEW SHEET FOR MATH 132 FINAL EXAM, FALL 2003

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Disclaimer: This review sheet serves to give a **highlight** of the topics covered after Midterm #2. It does NOT replace your textbook and/or your lecture notes.

Comments about the practice exams/homework:

- practice exams are on the course website — these are taken verbatim from old exams and may NOT cover the same materials as we do
- the practice exams are intended to give you an IDEA what the questions are like; your homework problems are indented to give you a chance to LEARN the course materials. The actual exam MAY contain problems DIFFERENT from those in the practice exams and/or homeworks!
- for additional practice: try the end-of-chapter review problems

Other comments about your exams:

- **SHOW YOUR WORK!**
- study the examples in your textbook
- the final exam is TWO HOURS LONG

11.1, 11.2

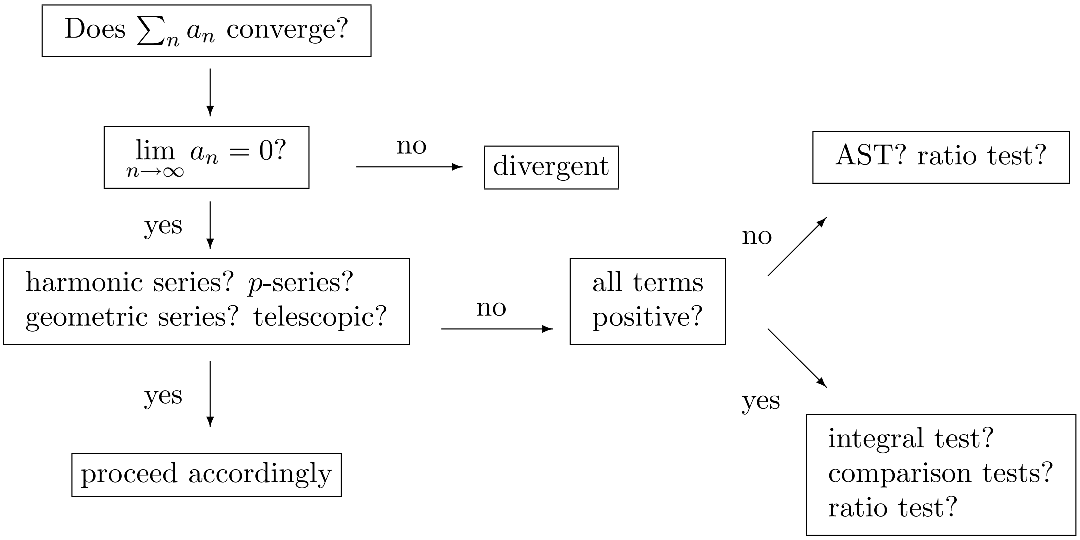
- a **series** $\sum a_n$ **converges** if and if only the **sequence of partial sums** $s_n = a_1 + \cdots + a_n$ converges
- **basic series:**

	harmonic series	geometric series	p-series	telescopic series
shape	$\sum_{n=1}^\infty \frac{1}{n}$	$\sum_{n=0}^\infty ar^n$	$\sum_{n=1}^\infty \frac{1}{n^p}$	Example: $\sum_{n=1}^\infty \frac{1}{n(n-2)}$
behavior	always divergent	conv if $\|r\| < 1$ div if $\|r\| \geq 1$	conv if $p > 1$ div if $p \leq 1$	

- **n-term test:**
  - if $\lim_{n\to\infty} a_n \neq 0$ or if this limit does not exist, then $\sum a_n$ diverges
  - **warning**: the n-term test is only a ONE-WAY test – if $\lim_{n\to\infty} a_n = 0$ then the series MAY or MAY NOT converge !!
- note the difference between a **p-series** $\boxed{\sum_n \frac{1}{n^p}}$ and a **geometric series** $\boxed{\sum_n \frac{a}{r^n}}$ — note the different locations where n appear

Date: May 12, 2003.

Flow-chart for convergent test:



11.3, 11.4

- integral test
  - suppose $a_n = f(n)$ for some continuous, **decreasing, positive** function f, then

$$\int_1^\infty f(x)\,dx \quad \text{converges} \quad \iff \sum a_n \text{ converges}$$

  - error estimate for integral test:

$$\int_{n+1}^\infty f(x)dx < s - s_n < \int_n^\infty f(x)dx$$

- comparison test
  - suppose both $\sum a_n, \sum b_n$ both have POSITIVE terms.
    - if $a_n \le b_n$ for all large n and if $\sum b_n$ converges, then $\sum a_n$ converges
    - if $a_n \ge b_n$ for all large n and if $\sum a_n$ diverges, then $\sum b_n$ diverges
  - error estimate for the comparison test: if $0 \le a_n \le b_n$ for all n and if $\sum_n b_n$ converges, then

$$0 \le s - s_n < b_{n+1} + b_{n+2} + \ldots$$

- limit comparison test
  - suppose both $\sum a_n, \sum b_n$ both have POSITIVE terms. If $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists and is finite **and non-zero**, then

$$\sum a_n \text{ converges} \iff \sum b_n \text{ converges}$$

  - NOTE: there is NO error estimate for the limit comparison test!
- to use the two comparisons we need something to compare! Good candidates: p-series and geometric series. Also, remember the **relative** size of functions:

$$\boxed{\text{powers of } \log n} \ll \boxed{\text{positive powers of } n} \ll \boxed{(\text{fixed number} > 1)^n} \ll \boxed{n!} \ll \boxed{n^n}$$

11.5:

- alternating series test If
  - every $b_i > 0$
  - $\lim_{n\to\infty} b_n = 0$
  - $b_1 \geq b_2 \geq b_3 \geq \cdots$

  then the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 \pm \cdots$ converges
- error estimates: suppose the alternating series $b_1 - b_2 + b_3 - b_4 \pm \cdots$ satisfies the AST, then

$$|s - s_n| \leq b_{n+1}$$

11.6:

- $\sum a_n$ is called **absolutely convergent** if $\sum |a_n|$ converges
- $\sum a_n$ is called **conditionally convergent** if $\sum a_n$ converges but $\sum |a_n|$ does not.
  - Equivalently: $\sum a_n$ converges but **not** absolutely converges
- Theorem: absolutely convergent $\Rightarrow$ convergent
  - converse is FALSE! E.g. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges but NOT absolutely converges.
- Ratio Test:

$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert$	exists and is < 1	exists and is > 1; **or** if the limit is $+\infty$	exists and is $= 1$; **or** if the limit does not exist
Conclusion	absolute conv.	divergence	inconclusive

11.8, 11.9

- given a power series, use the ratio test to find its radius of convergence, interval of convergence and its center
  - don't forget to check the end points when you try to determine the IOC !
- can get power series representation of functions by manipulating geometric series – the basic idea is to turn your expression into something that resemble $\boxed{\frac{1}{1 - \text{blah}}}$:
  - Step 0: make sure the numerator is 1
  - Step 1: make sure the denominator begins with the coefficient 1
  - Step 2: arrange the denominator so that it looks like $\boxed{1 - \text{blah}}$
  - Step 3: apply the geometric series formula – so long as $\boxed{|\text{blah}| < 1}$
- within the interval of convergence you can differentiate and/or integrate a power series term-by-term
  - use this to get power series representation for $\boxed{\arctan x}$ and for $\boxed{\ln(1 - x)}$ (both for $|x| < 1$)

11.10

- Taylor polynomials
- know how to compute Taylor series & MacLaurin series
  - Taylor series with center a: $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$
  - MacLaurin series: Taylor series with center 0; i.e. $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$
- know the series (and the interval of convergence) of the basic functions:

$$e^x,\ \sin x,\ \cos x,\ \arctan x,\ \ln(1-x),\ \frac{1}{1-x}$$

- applications: e.g. computing indefinite integrals; computing limits; estimating definite integrals (in conjunction with error estimates)

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