Sec 7.4 problem 76:

(a) Let n be a positive integer greater than or equal to 2.

$$I_{n-1} := \int \frac{dx}{(x^2+a^2)^{n-1}} =$$

$u = (x^2+a^2)^{1-n}$ $\quad u' = (1-n)(x^2+a^2)^{-n}(2x)$

$v = x$ $\quad v' = 1$

$$= \frac{x}{(x^2+a^2)^{n-1}} - (1-n)\int \frac{2x^2}{(x^2+a^2)^n}\,dx =$$

$2x^2 = 2(x^2+a^2) - 2a^2$

$$= \frac{x}{(x^2+a^2)^{n-1}} + 2(n-1)\left[\underbrace{\int \frac{dx}{(x^2+a^2)^{n-1}}}_{I_{n-1}} - a^2 \underbrace{\int \frac{dx}{(x^2+a^2)^n}}_{I_n}\right]$$

We get

$$I_{n-1} = \frac{x}{(x^2+a^2)^{(n-1)}} + 2(n-1) I_{n-1} - 2a^2(n-1) I_n$$

So

$$2a^2(n-1) I_n = \frac{x}{(x^2+a^2)^{n-1}} + (2n-3) I_{n-1}$$

$$\underbrace{I_n}_{\int \frac{dx}{(x^2+a^2)^n}} = \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} + \frac{2n-3}{2a^2(n-1)} \underbrace{I_{n-1}}_{\int \frac{dx}{(x^2+a^2)^{n-1}}}$$

case $a=1, n=2$ in part(a)

$$(b) \quad \int \frac{dx}{(x^2+1)^2} = \frac{x}{2\cdot 1^2\,(x^2+1)} + \underbrace{\frac{(2\cdot 2-3)}{2\cdot 1^2(2-1)}}_{\frac{1}{2}} \int \frac{dx}{x^2+1}$$

$$= \frac{x}{(x^2+1)} + \frac{1}{2}\arctan(x) + C$$

case $a=1, n=3$ in part(a)

$$\int \frac{dx}{(x^2+1)^3} = \frac{x}{2(3-1)(x^2+1)^2} + \underbrace{\frac{2\cdot 3-3}{2(3-1)}}_{\frac{3}{4}} \underbrace{\int \frac{dx}{(x^2+1)^2}}_{\parallel \text{ above}}$$

$$\left[\frac{x}{x^2+1} + \frac{1}{2}\arctan(x) + C'\right]$$

$$= \frac{x}{4(x^2+1)^2} + \frac{3}{4}\left[\frac{x}{x^2+1} + \frac{1}{2}\arctan(x)\right] + C.$$