Section 11.1 Problem 85:

$a_1 = \sqrt{2}$, $\quad a_{n+1} = \sqrt{2a_n}$.

(i) The sequence is bounded from above by 2.

Proof: (By induction that $a_n < 2$, for all $n \geq 1$).

$a_1 = \sqrt{2} < 2$, so the statement holds for a_1.
It suffices to show that if it holds for a_n, $n \geq 1$, then it also holds for a_{n+1}.
Assume that $a_n < 2$. Then

$$a_{n+1} = \sqrt{2a_n} \underset{\boxed{\text{since } a_n < 2 \text{, by assumption}}}{<} \sqrt{2 \cdot 2} = 2.$$

Hence $a_{n+1} < 2$ as well.

(ii) The sequence is increasing:

$$a_{n+1} = \sqrt{2a_n} \underset{\boxed{\text{by part (i)}}}{>} \sqrt{a_n a_n} = a_n.$$

(iii) $L := \lim_{n \to \infty} a_n$ exists, since $\underset{a_1}{\sqrt{2}} \leq a_n < 2$, for all n, by parts (i) and (ii), so $\{a_n\}$ is a bounded sequence, and it is increasing by part (ii) and so the hypotheses of the Monotonic Sequence Theorem are satisfied.

$$\sqrt{2} \leq L = \lim_{n \to \infty} a_n \underset{\text{Part (a) of Problem 3}}{=} \lim_{n \to \infty} a_{n+1} = \lim_{n \to \infty} \sqrt{2a_n} \underset{\boxed{\text{since } f(x)=\sqrt{x} \text{ is continuous at } L \text{ as } L \in (0,\infty).}}{=}$$

$$= \sqrt{2 \lim_{n \to \infty} a_n} = \sqrt{2L}.$$

Hence $L^2 = 2L$ and $L \geq \sqrt{2}$. Thus $\boxed{L = 2}$.