

Sec 11.1 page 736 # 53:

$$\lim_{n \rightarrow \infty} \underbrace{\left(1 + \frac{2}{n}\right)^n}_{a_n}$$

Let $b_n = \ln(a_n) = n \ln\left(1 + \frac{2}{n}\right) = \frac{\ln\left(1 + \frac{2}{n}\right)}{\left(\frac{1}{n}\right)}$

Then $b_n = \frac{\beta(x)}{g(x)}$, where $\beta(x) = \ln\left(1 + \frac{2}{x}\right)$ $g(x) = \frac{1}{x}$.

$$\lim_{x \rightarrow \infty} \ln\left(1 + \frac{2}{x}\right) = \ln(1) = 0$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

Thus, L'Hospital's Rule

yields: $\lim_{x \rightarrow \infty} \frac{\beta(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{\beta'(x)}{g'(x)} =$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{\left(1 + \frac{2}{x}\right)} \cdot \left(-\frac{2}{x^2}\right)}{-\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{2}{\left(1 + \frac{2}{x}\right)} = 2$$

Thus

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} e^{b_n} = e^{\lim_{n \rightarrow \infty} b_n} = e^2 = \boxed{e^2}$$

Sec 11.1 Theorem 4 +

e^x is continuous at $x=2$.

$$= \sqrt{2} \lim_{n \rightarrow \infty} a_n = \sqrt{2} L \Rightarrow L = 2L \Rightarrow \boxed{L=2}$$