Math 132H Hand written asssignment 1 Fall 2023

• It is not sufficient to just write the answers. You must *explain* how you arrive at your answers showing all your work.

1. Problem 98 of Section 5.5 page 427 (available on the Webassign e-book) Let f be a continuous function on the interval $[a, b]$. Use the substitution $u = \pi - x$ to show that
$$\int_0^\pi xf(\sin(x))dx = \frac{\pi}{2}\int_0^\pi f(\sin(x))dx.$$
Hint: Denote by L the left hand side and by R the right hand side. The suggested substitution will result in an equation: $L =$ [an expression involving L and R]. Now solve for L.

2. Problems Plus section of Chapter 5 page 434 problem 18 (available on the Webassign e-book). The figure shows all the points inside a square with edge length 2 that are closer to the center of the square than to the sides of the square. Find the area of the region. Hint: Show that the area of the region is given by
$$8\int_0^{\sqrt{2}-1}\left(\frac{1}{2} - \frac{x^2}{2} - x\right)dx$$
and evaluate the integral. Find first one eighth of the area lying in the first quadrant over the line $y = x$. You will need to find an equation for the upper boundary curve of the region. Let (x, y) be a point on this curve. Show that the distance from (x, y) to the center is equal to the distance from (x, y) to the upper edge of the square (use complete sentences and write a careful argument!). Express this equality as an equation in terms of x and y and solve for y as a function of x.

3. Section 6.1: Find the area of the region bounded by the left loop of the curve
$$y^2 = x^2(x + 3).$$
Include a graph and label the x and y coordinates of the leftmost and rightmost points on the loop, as well as an explicit equation of each bounding curve as a graph of some function). Hint:
$$x(x + 3)^{1/2} = (x + 3)(x + 3)^{1/2} - 3(x + 3)^{1/2}.$$

4. Section 6.2 problem 64 page 458 (available on the Webassign e-book): Find the volume of a pyramid with height h and base an equilateral triangle with side a. Support your answer with a sketch and a clear explanation.