Sec 5.5 page 415 #90; (~~18~~ 20 points)

$$I = \int_0^{\pi} x\, f(\sin(x))\, dx \underset{\substack{u=\pi-x \\ x=\pi-u \\ du=-dx}}{=} \int_{u=\pi}^{u=0} (\pi-u)\, f(\underbrace{\sin(\pi-u)}_{\substack{\parallel \\ \sin(u)}})\,(-du)$$

reverse end pts

$$= \int_{u=0}^{u=\pi} (\pi-u)\, f(\sin(u))\, du \underset{\substack{\uparrow \\ \text{rename variable of integration}}}{=}$$

$$= \pi \int_0^{\pi} f(\sin(x))\, dx - \underbrace{\int_0^{\pi} x\, f(\sin(x))\, dx}_{I}$$

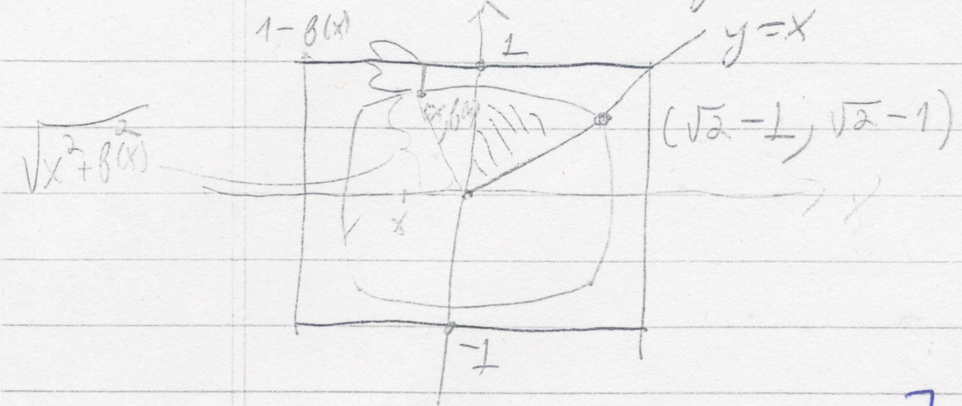
So, $$2I = \pi \int_0^{\pi} f(\sin(x))\, dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} f(\sin(x))\, dx,$$

Please return the solution to me.

Solution:

Problem Plus page 420 #16



Step 1: Equation of boundary: (~~6~~ 7 points)

Distance from edge $= 1-f(x)$

" " origin $= \sqrt{x^2+f^2(x)}$

$$(1-f(x))^2 = x^2 + f^2(x)$$

$$\parallel$$

$$1-2f(x)+f^2(x)$$

$$-2f(x) = x^2-1$$

$$f(x) = \frac{1}{2} - \frac{x^2}{2}$$

Step 2: (6 points)

Point of intersection of $y=f(x)$ and $y=x$:

$$\frac{1}{2} - \frac{x^2}{2} = x$$

$$x^2+2x-1=0 \qquad x_{12} = -1 \pm \sqrt{1-(-1)} = -1 \pm \sqrt{2}$$

Check $f(\sqrt{2}-1) = \frac{1}{2} - \frac{1}{2}(2-2\sqrt{2}+1) = -1+\sqrt{2}$ ✓

7

(~~6~~ points

Step 3: Area of region =

$$8\int_0^{\sqrt{2}-1}\left[\left(\frac{1}{2}-\frac{x^2}{2}\right)-x\right]dx =$$

$$= \frac{\cancel{8}}{4}\cdot\left[\frac{x}{2}-\frac{x^3}{6}-\frac{x^2}{2}\right]_0^{\sqrt{2}-1} =$$

$$x \quad -\frac{x^3}{3} \quad -x^2$$

$$= 4\left[(\sqrt{2}-1)-\frac{(\sqrt{2}-1)^3}{3}-(\sqrt{2}-1)^2\right] =$$

There is no need to simplify

$$= 4(\sqrt{2}-1)\left[1-\frac{(\sqrt{2}-1)^2}{3}-(\sqrt{2}-1)\right]$$

$$2-\frac{2-2\sqrt{2}+1}{3}-\sqrt{2}$$

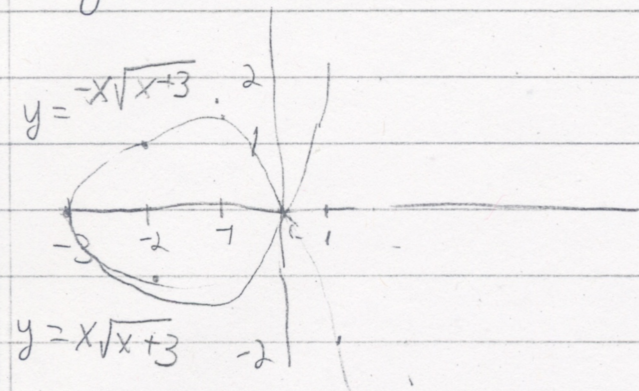
$$2-1+\frac{2}{3}\sqrt{2}-\sqrt{2}$$

$$\left(1-\frac{2}{3}\sqrt{2}\right)$$

$$1-\frac{1}{3}\sqrt{2}$$

Sec 6.1 page 428 # 49

$y^2 = x^2(x+3)$



$$\text{Area} = \int_{-3}^{0} \overbrace{-x\sqrt{x+3}}^{\text{high}} - \overbrace{x\sqrt{x+3}}^{\text{low}} \, dx$$

$$= -2\int_{-3}^{0} \underbrace{x\sqrt{x+3}}_{(x+3)\sqrt{x+3} \; - \; 3\sqrt{x+3}} \, dx =$$

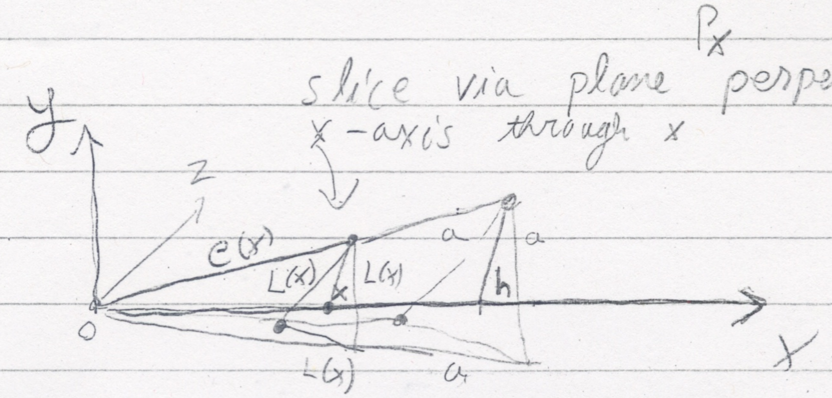
$$\underbrace{(x+3)\sqrt{x+3}}_{(x+3)^{3/2}}$$

$$-2\left[\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2}\right]_{-3}^{0} =$$

$$= -2\left(\underbrace{\frac{2}{5}3^{5/2}}_{\frac{6}{5}3^{3/2}} - 2\cdot 3^{3/2}\right) = \frac{8}{5}\cdot 3^{3/2}$$

$$\left(-\frac{4}{5}\right)3^{3/2}$$

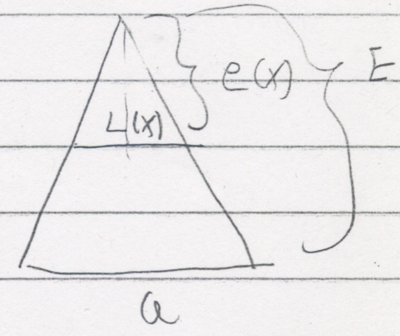
Sec 6.2 page 440 #52



$L(x)$ = length of triangle edge.

The planes P_x and P_h are parallel and so they intersect the faces of the pyramid along parallel lines.

It follows that the triangles on the face of the pyaramid are similar, and



$$\frac{L(x)}{a} = \frac{e(x)}{E}$$

Above we chose the x-axis to pass though the mid point of the base.

We claim that $\boxed{\frac{L(x)}{a} = \frac{x}{h}}$.

For this we a construction of two similar (red) triangles with edges of length: small triangle: $e(x)$, x; large: E, h

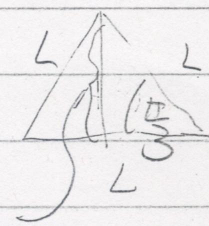
to get $\frac{e(x)}{E} = \frac{x}{h}$.

We conclude that $L(x) = \frac{ax}{h}$.

The area of the cross-section is

$$A(x) = \frac{1}{2}\underbrace{(\text{base})}_{L(x)}\underbrace{(\text{height})}_{\frac{\sqrt{3}}{2}L(x)}$$

$$= \frac{\sqrt{3}}{4}\left(L(x)\right)^2 = \frac{\sqrt{3}}{4}\frac{a^2}{h^2}x^2$$



$$\frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right) = \frac{\text{height}}{L}$$

$$\text{Vol} = \int_0^h \frac{\sqrt{3}}{4}\frac{a^2}{h^2}x^2\,dx$$

$$= \frac{\sqrt{3}}{4}\frac{a^2}{h^2}\left[\frac{x^3}{3}\right]_0^h = \frac{\sqrt{3}}{4}\frac{a^2}{h^2}\frac{h^3}{3} =$$

$$\boxed{\text{Vol} = \frac{a^2h}{4\sqrt{3}}}$$