MATH 132H FALL 2012 FINAL EXAM

Your Name: _____

This is a two hours exam. This exam paper consists of 7 questions. It has 9 pages.

On this exam, you may use a calculator and one letter size page of notes, but no books.

It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

1. (14)   _____

2. (14)   _____

3. (14)   _____

4. (14)   _____

5. (14)   _____

6. (18)   _____

7. (18)   _____

TOTAL (106)

1. (14 points) Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n(5n+1)}$.
   Justify your answer (do not forget to justify the convergence or divergence at the endpoints).

2. (14 points) a) Find the Maclaurin series for $f(x) = \dfrac{1}{1+x^2}$. Justify your answer!

   b) Use your answer in part (a) in order to find the Macluarin series of $\tan^{-1}(x)$. Prove your answer.

3. (14 points) Use Taylor's Inequality and the Maclaurin series $e^x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ in order to find the minimal n, such that the n-th Taylor polynomial $T_n(x)$ approximates e^x with an error $\leq 10^{-5}$ in the interval $-1 \leq x \leq 1$.

4. (14 points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. Explain which test you used and why all the conditions of the test are satisfied.

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$.

   b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n} + \ln(n)}{1+n^2}$.

5. (14 points) Compute the following integrals **algebraically**. Show all your work!

   a) $\displaystyle\int_1^{\infty} \frac{dx}{\sqrt{x}(\sqrt{x}+1)(\ln(\sqrt{x}+1))^2} =$

   b) $\displaystyle\int \cos(x)e^{2x} dx =$

6. (18 points) Consider the curve given by the parametric equations $x = e^{-t}\cos(t)$, $y = e^{-t}\sin(t)$, $0 \leq t \leq 2\pi$.
   a) Find all the point where the tangent line to the curve is horizontal and all the point where the tangent line is vertical. Show all your algebraic steps.

   b) Sketch the graph of the portion of the curve for $0 \leq t \leq \pi$. Indicate the polar coordinates of the points of intercept with the x and y axis, the scale, and the horizontal and vertical tangent lines and the polar coordinates of the points with these tangent lines. (The portion of the curve in the interval $\pi < t \leq 2\pi$ will be too small to draw).

   c) Find the length of the curve in part (a). Show all your algebraic steps.

7. (18 points) a) Find the cartesian equation of the polar curve $r = 5\sin(\theta)$.

   b) Sketch the region that lies inside the polar curve $r = 5\sin(\theta)$, from part (a), and outside the polar curve $r = 2 + \sin(\theta)$. Provide polar coordinates for all points of intersection.

   c) Find the area of the region in part (b).