MATH 132H FALL 2012 EXAM 2

Your Name: ________________________________

This is a 90 minutes exam. This exam paper consists of 6 questions. It has 9 pages.

On this exam, you may use a calculator and one letter size page of notes, but no books.

It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

1. (10 points) Evaluate the integral $\int_0^1 \frac{x}{x^2+5x+6}dx$ using partial fractions. Show all your algebraic steps.

2. a) (8 points) Use the comparison test in order to determine if the following improper integral is convergent or divergent $\int_1^\infty \frac{\ln(x)}{x^2+1}\,dx$. Carefully justify your answer!

   b) (8 points) Evaluate the improper integral showing all your algebraic steps $\int_0^1 \frac{e^{(-1/x)}}{x^3}\,dx$

3. (14 points) For each of the following **sequences** (not series) determine whether the sequence converges or diverges. If it converges, find the limit, showing all your algebraic steps. Otherwise, explain why it diverges.

   a) $a_n \;=\; \sqrt{n+2}-\sqrt{n}, \quad n \geq 1.$
   Hint: Use the identity $(a-b)(a+b) = a^2 - b^2$.

   b) $a_n \;=\; (n^2+3)^{1/n}, \quad n \geq 1.$

4. (14 points)

   (a) Find the values of x, for which the series $\sum_{n=0}^{\infty} \frac{(2x-3)^n}{9^n}$ converges. State the convergence test you use and explain why its hypotheses are satisfied.

   (b) Find the sum of the series for those values of x. Simplify your answer.

5. (14 points) Let s be the sum of the series $\sum_{n=1}^{\infty} \frac{4}{n^5}$ and s_n the n-th partial sum. Find the **minimal** number n of terms of the series, for which we know that $s - s_n \leq 10^{-8}$, by the error estimate of the integral test. Justify your answer, showing all your algebraic steps.

6. (32 points) Determine whether the following series converge absolutely, converge conditionally, or diverge. Name each test you use and indicate why all the conditions needed for it to apply actually hold.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+10}}{n^2+3n+5}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n e^{1/n}$

   (c) $\displaystyle\sum_{n=3}^{\infty} (-1)^n \left(\frac{\ln(n)}{n}\right)$

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n!2^n}{(2n)!}\right)$