MATH 132 FALL 2009 FINAL EXAM

1. Evaluate the following integrals. Explicitly show any relevant algebraic manipulation.

   a)(7 points) $\int_0^2 (x^2+1)e^x dx$

   b) (8 points) $\int \sqrt{1-4x^2}dx$

2. (10 points) Find the volume of the infinite solid of revolution obtained by rotating the curve $y = \left(\frac{1}{x}\right)^{2/3}$ around the x-axis, over the interval $[1, \infty)$. Carefully justify your answer

3. (a) (5 points) Set-up a definite integral for the total length of the ellipse, given as the parametrized curve $x = 2\cos(\theta)$, $y = 3\sin(\theta)$, $0 \le \theta \le 2\pi$. Do **not** evaluate the integral.

   (b) (6 points) Sketch the region in the first quadrant that lies inside the polar curve $r = 2\sin(2\theta)$ and outside the polar curve $r = \sqrt{2}$. Provide polar coordinates for all points of intersection in the first quadrant.

   (c) (8 points) Determine the area of the region in part 3b.

4. a) (5 points) Find the derivative of the function $G(x) := \int_2^{1/x} \arctan(t)dt$

   b) (7 points) The velocity function, in meters per seconds, for a particle moving along a line is $v(t) = t^2 - 2t - 8$. Find the distance traveled by the particle (**not** the displacement) during the time interval $2 \le t \le 6$.

5. Determine whether the following series converge absolutely, converge conditionally, or diverge. Name each test you use and indicate why all the conditions needed for it to apply actually hold.

   (a) (7 points) $\sum_{n=1}^{\infty} (-1)^n \frac{2n^2}{7n^4 - n^3 + 1}$

   (b) (7 points) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln(n))^2}$

6. a) (7 points) Find the Maclaurin series for $f(x) = \ln(1+3x)$.

   b) (8 points) Determine the **interval** of convergence of the series $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{n}$. Justify your answer with calculations. Do not forget to check for convergence at the end points.

7. a) (7 points) Let $f(x) = e^x + e^{-x}$ and denote by $T_n(x)$ its Taylor polynomial, centered at 0, involving powers of x of degree $\le n$. Find the Taylor polynomial $T_7(x)$.

   b) (8 points) Use Taylor's Inequality to show that the error $|f(x) - T_7(x)|$, of approximating $f(x)$ by $T_7(x)$, is bounded by 0.0001 over the interval
   $-1 \le x \le 1$. Carefully justify your answer!