MATH 132 FALL 2009 EXAM 2

1. a) (12 points) Determine for which positive real number p, does the following improper integral converges, and for which it diverges. Evaluate the integral for each value of p, for which it converges, and express its value in terms of p. Justify your answer and show all your algebraic steps. Hint: Do not forget to consider the case $p = 1$ as well.
   $\int_e^\infty \frac{1}{(\ln(x))^p x}\, dx$

   b) (10 points) Determine whether the following improper integral converges or diverges. Evaluate it, showing all your algebraic steps, if it is convergent. Otherwise, explain why it is divergent.
   $\int_0^8 \frac{1}{(x-8)^{(2/3)}}\, dx$

2. (14) For each of the following **sequences** (not series) determine whether the sequence converges or diverges. If it converges, find the limit, showing all your algebraic steps. Otherwise, explain why it diverges.

   a) $a_n \;=\; \frac{\sqrt{2n^2+3}}{3n-1}, \quad n \geq 1.$

   b) $a_n \;=\; n\sin(n)e^{-n}, \quad n \geq 1.$

3. Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^6}$.

   (a) (9 points) Use the integral test to show that the series is convergent. Show that all the hypothesis of the test are satisfied. Show all your algebraic steps. (Credit will not be given for an answer using another test).

   (b) (9 points) Let s be the sum of the series in part 3a. Find the **minimal** number n of terms of the series, for which we know that $s - s_n \leq 0.00001$, by the error estimate of the integral test. Justify your answer, showing all your algebraic steps.

4. (14 points) Consider the series $\sum_{n=1}^{\infty} \frac{5}{3+2^n}$

   (a) (5 points) Use the comparison test to show that the series converges.

   (b) (9 points) The sum s_9 of the first 9 terms of the series, rounded to five decimal digits, is 2.71665. You do not need to verify this. Show that $s - 2.71665$ is less than 0.01. Justify your answer!

5. (32 points) Determine whether the following series converge absolutely, converge conditionally, or diverge. Name each test you use and indicate why all the conditions needed for it to apply actually hold.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+5}{5n^3 - 2n^2 + 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{4^n}{n!}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n} \left(\frac{2n^2+n}{3n^2+7n}\right)^{2n}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n} \left(\frac{n+2}{n^2+4}\right)$