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Huybrechts, Prob 1.1.6 page 23:
Change variables to $z = z_2$, $w = z_1$
The function

$$f(z,w) = w^3 z + wz + w^2 z^2 + z^2 + wz^3 =$$
$$= z\left(w^3 + w + w^2 z + z + wz^2\right)$$

vanishes identically on the w-axis, but not on the z-axis, so we will use WPT to find

$f = gh$ with $g(z,w)$ a poly in z.

$f(z,0) = z^2$, so g should have degree 2 in z.

Set $\bar{f}(z,w) = w^3 + w + w^2 z + z + wz^2 = \dfrac{f(z,w)}{z}$,

Then $\bar{f}(z,w)$ should have a factorization as in WPT

$\bar{f} = \bar{g} h$ with the same h, but $g = z\bar{g}$.

So $\bar{g} = z + a(w)$, where $a(w) = \sum_{n=1}^{\infty} a_1 w^n$ holo at 0.

Now $\bar{f}_w(z) = wz^2 + (w^2+1)z + (w^3+w)$ is a quadratic poly in z, while $\bar{g}$ has degree one in z. So $\bar{g}_w(z)$ should vanish along only one of the two roots in the factorization $\bar{f}_w(z) = w(z - \lambda_1(w))(z - \lambda_2(w))$,

for w fixed and sufficiently small.

Using the quadratic formula we find

$$\lambda_{1,2} = \frac{-(1+w^2) \pm \sqrt{(1+w^2)^2 - 4w(w^3+w)}}{2w}$$

where we take the branch of the square root with value 1 at $w=0$.

So

$$B_w(z) = \Bigg(\underbrace{z - \left[\frac{-(1+w^2) \pm \sqrt{(1+w^2)^2 - 4w(w^3+w)}}{2w}\right]}_{\bar{g}(z,w)}\Bigg) \cdot$$

$$\Bigg(\underbrace{wz + \frac{1}{2}\left[(1+w^2) + \sqrt{(1+w^2)^2 - 4w(w^3+w)}\right]}_{h(z,w)}\Bigg)$$

Note that $h(0,0) = 1 \neq 0$, so indeed h does not vanish at $(0,0)$.

$$g(z,w) = z \cdot \bar{g}(z,w) = z^2 - z\left[\frac{-(1+w^2) + \sqrt{(1+w^2)^2 - \cdots}}{2w}\right]$$