2.3.7 $\quad x \in \mathbb{P}^n = \mathbb{P}(V) \quad V \simeq \mathbb{C}^{n+1}$

a) $W := \{ s \in \underbrace{H^0(\mathbb{P}^n, \mathcal{O}(1))}_{V^*} : s(x) = 0 \}$

Let $l_x \subset V$ be the line corr to the point x. Then $W = (V/l_x)^* \subset V^*$

$$\varphi : \mathbb{P}(V) \setminus \{x\} \to \mathbb{P}(\underbrace{V/l_x}_{W^*})$$

sends $y \in \mathbb{P}(V) \setminus \{x\}$ to the point $\overbrace{l_x + l_y}^{\text{one-dim'l}} / l_x \in \mathbb{P}(V/l_x)$.

b) In coordinates, we may assume that $x = (0 : \dots : 0 : 1)$, so $W = \mathrm{span}\{z_0, \dots, z_{n-1}\}$ and

$$\varphi : \mathbb{P}^n \setminus \{x\} \to \mathbb{P}^{n-1} \quad \text{sends}$$

$$(z_0 : \dots : z_n) \text{ to } (z_0 : \dots : z_{n-1})$$

PROJECTION (from the point x).

c) If we choose a hyperplane $H \subset \mathbb{P}^n$ not passing through x (e.g., $H := (z_n = 0)$ in description b) then φ restricts to H as an isomorphism onto $\mathbb{P}^{n-1}$ and we may regard φ as a map $\varphi : \mathbb{P}^n \setminus \{x\} \to H$ sending y to the point of intersection of the line $l_{x,y}$ through x and y with H