Huybrechts, page 64 problem 2.1.4:

Let X be a simply connected compact complex manifold. Then any holomorphic map $f : X \to T$, where $T = \mathbb{C}^n / \Gamma$ is a compact complex torus, is constant.

Proof: Step 1: Let $q : \mathbb{C}^n \to T$ be the universal cover (mapping z to $z + \Gamma$). There exists a continuous lift $\tilde{f} : X \to \mathbb{C}^n$, such that $q \circ \tilde{f} = f$, since X is simply connected (the map lifting property of the universal cover).

Step 2: $\tilde{f}$ is holomorphic: The map q is holomorphic, by definition of the complex structure of T, and q is locally invertible, i.e., for each point $P \in T$ there exists a connected open neigh U of P, such that for every connected component $\tilde{U} \subset \mathbb{C}^n$ of $q^{-1}(U)$, the restriction $q|_{\tilde{U}} : \tilde{U} \to U$ is an isomorphism. So if V is a connected component of $f^{-1}(U)$, then

$$\tilde{f}|_V : V \to \mathbb{C}^n \text{ is the composition}$$

of $q^{-1}_{|\tilde{U}} \circ f$, where $\tilde{U}$ is the connected component of $q^{-1}(U)$ containing $\tilde{f}(V)$. Thus, $\tilde{f}_{|V}$ is the composition of holomorphic maps, hence holomorphic. Thus, $\tilde{f}$ is holomorphic everywhere.

Step 3: Write $\tilde{f} := (\tilde{f}_1, \dots, \tilde{f}_n)$, where $\tilde{f}_i : X \to \mathbb{C}$ is holomorphic. Then $\tilde{f}_i$ is a constant function, for $1 \le i \le n$, since X is compact. Hence $\tilde{f}$ is a constant function. Thus $f := g \circ \tilde{f}$ is a constant function.

Q.E.D.