Huybrechts' page 50, problem 1,3,5

$$\omega = \frac{i}{2\pi}\partial\bar\partial \log(|z|^2+1) \in A^{1,1}(\mathbb{C}).$$

Then ω is a $(1,1)$-form, by construction, and closed, since $\dim_{\mathbb{R}}(\mathbb{C}) = 2$.

A metric g on $\mathbb{C}$, for which I is compatible, is determined by one positive function $h(z) := g_z(\frac{\partial}{\partial x}, \frac{\partial}{\partial x})$, since then, for all $z \in \mathbb{C}$,

$$g_z(\tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial y}) = g_z(\tfrac{\partial}{\partial x}, I(\tfrac{\partial}{\partial x})) = 0$$

$$g_z(\tfrac{\partial}{\partial y}, \tfrac{\partial}{\partial y}) = g_z(I(\tfrac{\partial}{\partial x}), I(\tfrac{\partial}{\partial x})) = g_z(\tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial x}) = h(z).$$

Furthermore, in that case the fundamental form is

$$\omega = \frac{i}{2} h \, dz \wedge d\bar z,$$

(osculates to the standard metric in some coordinates)

and it is Kähler, by Prop. 1.3.12.

So it remains to check that

$$h(z) = \frac{1}{\pi}\frac{\partial}{\partial z}\frac{\partial}{\partial \bar z}\log(|z|^2+1)$$ is positive everywhere.

$$\frac{\partial}{\partial \bar z}\log(|z|^2+1) = \frac{\frac{\partial}{\partial \bar z}(z\cdot\bar z+1)}{|z|^2+1} = \frac{z}{|z|^2+1}$$

$$\frac{\partial}{\partial z}\left(\frac{z}{|z|^2+1}\right) = \frac{(|z|^2+1) - z\overbrace{\frac{\partial}{\partial z}(z\cdot\bar z+1)}^{=\bar z}}{(|z|^2+1)^2} = \frac{1}{(|z|^2+1)^2} > 0.$$

$\square$