Huybrechts' page 50 problem 1.3.3

Let B be a polydisk in $\mathbb{C}^n$.
Let $\alpha \in A^{p,q}(B)$ d-closed, $p,q \geq 1$.
Then there exists $\gamma \in A^{p-1,q-1}(B)$, such that $\partial\bar\partial\gamma = \alpha$.

Proof: There exists a $(p+q-1)$-form β, such that $d\beta = \alpha$, by the d-Poincaré Lemma.
Let $\beta' \in A^{p-1,q}(B)$, $\beta'' \in A^{p,q-1}$ be the projections of β. If $(r,s) \notin \{(p-1,q),(p,q-1)\}$, then the projection of β to $A^{r,s}(B)$ is closed, and so $d\beta = d(\beta'+\beta'')$, since α is of type (p,q). Hence, $\bar\partial\beta' = 0$, $\partial\beta'' = 0$ and $\boxed{\partial\beta' + \bar\partial\beta'' = \alpha.}$
The $\bar\partial$-Poincaré Lemma and its ∂-analogue imply that there exist $\gamma', \gamma'' \in A^{p-1,q-1}(B)$, such that $\boxed{\beta' = \bar\partial\gamma'}$ and $\boxed{\beta'' = \partial\gamma''.}$
Take $\gamma := \gamma' - \gamma''$.
Then

$$\partial\bar\partial(\gamma) = \partial(\bar\partial\gamma') - \underbrace{\partial\bar\partial}_{= -\bar\partial\partial}(\gamma'') =$$

$$= \partial\beta' + \bar\partial\beta'' = \alpha.$$

QED