1.2.5 page 40: $\mathbb{H} = \mathbb{R} + \mathbb{R}I + \mathbb{R}J + \mathbb{R}K$, the quaternions algebra, considered as a subalgebra of $\mathrm{End}_{\mathbb{R}}(V)$.

Step 1: Let $(,)$ be the inner product of V. We show that V is an orthogonal direct sum of 4-dimensional subspaces, each $\mathbb{H}$-invariant, and each isomorphic to $\mathbb{H}$ as an $\mathbb{H}$-module.

Choose a vector $v_1 \in V$, satisfying $(v_1, v_1) = 1$.
$(v_1, Iv_1) = (Iv_1, \underbrace{I^2}_{-1} v_1) = -(v_1, Iv_1)$. So $(v_1, Iv_1) = 0$.

Similarly, $(v_1, Jv_1) = 0 = (v_1, Kv_1)$.
Now, $(Iv_1, Jv_1) = (Iv_1, K(Iv_1)) = 0$, since K is a CACS,
$(Iv_1, Kv_1) = (Iv_1, -J(Iv_1)) = 0$, since $-J$ is a CACS.
Finally, $(Jv_1, Kv_1) = (Jv_1, I(Jv_1)) = 0$, since I is a CACS.
So $\Sigma := \{v_1, Iv_1, Jv_1, Kv_1\}$ is an orthonormal set, hence linearly independent. Let W be the subspace spanned by Σ. Then W is $\mathbb{H}$-invariant and the linear transformation $\beta : \mathbb{H} \to W$, defined by $\beta(1) = v_1$, $\beta(I) = Iv_1$, $\beta(J) = Jv_1$, $\beta(K) = Kv_1$ is an $\mathbb{H}$-module isomorphism. The subspace $W^{\perp} \subset V$ is $\mathbb{H}$-invariant, since I, J, and K are isometries.
The statement follows, by induction on $\dim_{\mathbb{R}}(V)$.

The unit purely imaginary quaternions $\{aI + bJ + cK : a^2 + b^2 + c^2 = 1\}$ all act as compatible almost complex structures on V. We see a 2-sphere of such CACS's.

We prove that $\omega_J + i\omega_K$ is a $(2,0)$-form.

Step 2: Let $(V, (\,,\,), I)$ be an inner product space with a compatible almost complex structure. Let $\varphi_\theta : V \to V$ be the endomorphism acting on V by $\cos(\theta)\cdot id_V + \sin(\theta)\cdot I$. Then

$V^{1,0}$ is the $e^{i\theta}$- eigenspace of φ_θ and

$V^{0,1}$ " " $e^{-i\theta}$- " .

So $\overset{2}{\wedge}\varphi_\theta$ acts on $\overset{2}{\wedge}V$ and

$V^{2,0}$ is the $e^{i2\theta}$ - eigenspace

$V^{1,1}$ " " 1 - " , and

$V^{0,2}$ " " $e^{-i2\theta}$ - " , provided

$e^{2\theta i} \notin \{1, -1\}$.

Set $\omega_I := (I(\cdot), (\cdot))$, $\omega_J := (J(\cdot), (\cdot))$, $\omega_K := (K(\cdot), (\cdot))$.

Then

$$\omega_J(\varphi_\theta(x), \varphi_\theta(y)) = (\underbrace{J\circ(\cos(\theta) id_V + \sin(\theta) I)}_{(\cos(\theta) id_V - \sin(\theta) I) J(x)}(x), (\cos(\theta) id_V + \sin(\theta) I)(y))$$

$$= \cos^2(\theta)\,\omega_J(x,y) - \sin^2(\theta)\,\omega_J(x,y) +$$

$$+ \cos(\theta)\sin(\theta)\underbrace{(J(x), I(y))}_{-(IJ(x),y)} - \cos(\theta)\sin(\theta)(\underbrace{IJ}_{K}(x), y)$$

$$= \cos(2\theta)\,\omega_J(x,y) - \sin(2\theta)\,\omega_K(x,y)$$

Similarly,

$$\omega_K(\varphi_\theta(x), \varphi_\theta(y)) = \cos(2\theta)\, \omega_K(x,y) - \sin(2\theta)\, \omega_{\underbrace{IK}_{-J}}(x,y),$$

$$= \cos(2\theta)\, \omega_K(x,y) + \sin(2\theta)\, \omega_J(x,y).$$

So, $(\omega_J + i\,\omega_K)(\varphi_\theta(x), \varphi_\theta(y)) =$

$$\left[\cos(2\theta)\, \omega_J(x,y) - \sin(2\theta)\, \omega_K(x,y)\right] + i\left[\cos(2\theta)\, \omega_K(x,y) + \sin(2\theta)\, \omega_J(x,y)\right]$$

$$= \underbrace{\left[\cos(2\theta) + i\sin(2\theta)\right]}_{e^{2i\theta}} \left[\omega_J(x,y) + i\,\omega_K(x,y)\right].$$

Hence, $(\omega_J + i\,\omega_K)$ is an $e^{2i\theta}$-eigenvector of $\overset{2}{\Lambda}\varphi_\theta^*$ and so a $(2,0)$-form. Q.E.D