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Huybrechts 1.2.1 page 40:
$(V, (\,,\,))$ four dimensional $\mathbb{R}$-vector space with inner product. The set of all compatible almost complex structures consists of two copies of S^2.

Proof: Choose $v_1 \in V$, $(v_1, v_1) = 1$. Set $v_2 := I v_1$. Choose $v_3 \in \{v_1, v_2\}^\perp$ with $(v_3, v_3) = 1$. Set $v_4 := I(v_3)$. Then $\mathcal{B} := \{v_1, v_2, v_3, v_4\}$ is an orthonormal basis for V and the $\mathcal{B}$matrix of I is

$$[I]_{\mathcal{B}} = \left(\begin{array}{cc|cc} 0 & -1 & & \\ 1 & 0 & & \\ \hline & & 0 & -1 \\ & & 1 & 0 \end{array}\right).$$

$O(V)$ acts transitively on the set of orthonormal bases for V. Hence, every two complex structures are conjugate in $O(V)$.
Now $O(V)$ has two connected component, $SO(V)$, and orthogonal matrices with determinant -1. Hence, the set of all compatible complex structures has at most two connected components.

The stabilizer of I in $SO(V)$ is 4 dimensional, since the tangent space to the stabiliser of $[I]_{\mathcal{B}}$ consists of skew-symmetric matrices which commute with $[I]_{\mathcal{B}}$ so of the form $\begin{pmatrix} A & B \\ -B & D \end{pmatrix}$, A, D skew symm, and $B = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is the matrix of mult by a complex number.

Hence, each $SO(V)$ conjugacy class is 2-dim'l $2 = \underbrace{\dim(SO(V))}_{6} - 4$.

Finally, take $V = \mathbb{H} = \mathrm{Sp}_{\mathbb{R}}\{1, i, j, k\}$

$(q_1, q_2) = q_1 \circ \bar{q}_2$ (where $\bar{1} = 1$, $\bar{i} = -i$, $\bar{j} = -j$, $\bar{k} = -k$)

and observe that each unit purely imaginary quaternion $q = ai + bj + ck$, $a^2+b^2+c^2$, satisfies $q^2 = -1$, so both left multiplication L_q of $\mathbb{H}$ by q and right " R_q of $\mathbb{H}$ by q are almost complex structures. Writing their matrices, we see that $R_q \neq L_{q'}$ for every such q, q'. So we have two orbits, each an S^2.

Remark; The Pfaffian of a skew-symmetric 4x4 matrix M is $\mathrm{Pf}\underbrace{\begin{pmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{pmatrix}}_{M} = af - be + dc$. It is an invariant poly w.r.t the $SO(4)$ conjugation action and $\det(M) = \mathrm{Pf}(M)^2$. The matrices $[I]_{\mathcal{B}}$ w.r.t an orthonormal basis $\mathcal{B}$ are skew-symmetric, and the two $SO(V)$ orbits are distinguished by their Pfaffian (it has values 1 or -1).

Indeed, if $\beta = \{1, i, j, k\}$, and $q = ai + bj + ck$
$a^2 + b^2 + c^2 = 1$,

then

$$[L_q]_{\beta} = \begin{pmatrix} 0 & -a & -b & -c \\ a & 0 & -c & b \\ b & c & 0 & -a \\ c & -b & a & 0 \end{pmatrix} \quad \text{and } Pf(L_q) = 1$$

$$[R_q]_{\beta} = \begin{pmatrix} 0 & -a & -b & -c \\ a & 0 & c & -b \\ b & -c & 0 & a \\ c & b & -a & 0 \end{pmatrix} \quad \text{and } Pf(R_q) = -1.$$